OPTIMIZATION OF A CLOSED BRAYTON CYCLE FOR NAVY SHIP CONCEPTUAL DESIGN

Gary Lee Kavanagh

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ABSTRACT

The thermodynamic potential of the closed Brayton cycle has been well recognized for many years. This power cycle has been proposed for application to Navy ship propulsion because of its attributes of higher efficiencies and lower weight and volume requirements than existing cycles. Although the closed Brayton cycle has not yet seen Naval application except in small research units, the open Brayton cycle has been accepted and installed in new Navy destroyer ship classes.

One of the primary goals of Navy conceptual ship design is to develop a concept within acquisition cost constraints. Since ship displacement is often used as surrogate for acquisition cost, and consistent with the primary goals of conceptual design, the primary operating variables for the power cycle can be chosen so as to reduce system weight. One of these variables is total cycle design pressure loss ($\triangle P/P$), which is allocated to the system heat exchangers for design. Pressure drop is a significant design parameter for heat exchangers, but not for other system components; consequently, for given cycle operating conditions, allocation of the total cycle design pressure loss to the heat exchangers in such a manner as to minimize the total heat exchanger package weight, will also minimize the total system weight.

The primary purpose of the study is to derive such an optimum pressure loss allocation method. First, a description and thermodynamic analysis for the closed Brayton cycle is presented. Second, an optimum pressure loss allocation method is derived for a regenerative closed

cycle with shell-and-tube heat exchangers. It is found that the heat exchanger weight is proportional to the tube pressure drop to the -0.41 power, and, employing the LaGrange multiplier technique, an optimum allocation procedure is derived. Next, the system weight and performance impacts of each of the heat exchanger sizing and cycle operating parameters is evaluated. Tube diameter and diameter ratio are found to have significant impact upon weight, and tube spacing is seen to be important to design flexibility. It is determined that the highest cycle pressure level and turbomachinery efficiencies and inlet temperatures will promote higher cycle performance and lower weight. Design trade-offs are identified and examined for compressor pressure ratio, regenerator effectiveness and total cycle design pressure loss. From the analysis it is concluded that, in order to minimize the total system weight, the designer should prudently select the cycle operating and heat exchanger sizing parameters to take advantage of the weight and performance effects for the cycle configuration and power level required. Based upon these selected values, the optimum allocation method could then be used to distribute the cycle design pressure loss for the minimum system weight.

Thesis Supervisor: A. Douglas Carmichael

Title: Professor of Power Engineering

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NOMENCLATURE

Thermodynamic Calculations:

C_p .... Specific heat (Btu/lb_m-^oR)
H .... Enthalpy (Btu/lb_m)
HV .... Fuel heating value (Btu/lb_m)
P .... Absolute pressure (psia)
$\Delta P/P$ ...... Total cycle design pressure loss (%)
Q .... Heat energy transferred (Btu/sec)
sfc .... Specific fuel consumption (lb_m fuel/HP-hr)
T .... Absolute temperature (^oR)
w .... Mass flow rate (lb_m/sec)
W_c .... Compressor work (HP)
W_t .... Turbine work (HP)
W_T .... Cycle net work (HP)
ϵ_R .... Regenerator effectiveness (%)
η .... Cycle thermal efficiency (%)
η_b .... Combustion loop efficiency (%)
η_c .... Compressor efficiency (%)
η_t .... Turbine efficiency (%)
γ .... Specific heat ratio

Heat Exchanger Calculations:

A .... Heat transfer surface area (ft^2)

A_x .... Flow cross sectional area (ft^2)

D_0 .... Tube outside diameter (ft)

D_1 .... Tube inside diameter (ft)

D_2 .... Shell equivalent hydraulic diameter (ft)

D_s .... Heat exchanger shell diameter (ft)

D_r .... Tube diameter ratio (D_0/D_1)

G .... Mass velocity (lb_m/ft^2-sec)

g_o .... Conversion factor = 32.2 lb_m-ft/lb_f-sec^2

h .... Heat transfer coefficient (Btu/ft^2-sec-^oR)

L .... Heat exchanger tube length (ft)

n .... Number of heat exchanger tubes

ΔP .... Pressure drop (psi)

$\Delta P/P$ ...... Pressure loss (%)

P_H .... Cycle high pressure level (psia)

P_L .... Cycle low pressure level (psia)

Pr .... Prandtl number = $C_p\mu/k$

P* .... Design pressure = 1.5P (psia)

q .... Hydraulic diameter coefficient = D_2/D_0

Re .... Reynold's number = GD/μ

s/D_0 .... Tube spacing ratio

ΔT_{lm} ...... Log mean temperature difference (^oR)- defined by equation (6)

U .... Overall heat transfer coefficient (Btu/ft^2-sec-^oR)

t_s .... Heat exchanger shell thickness (ft)

W .... Total heat exchanger package weight (lb_m)

W_C .... Cooler weight (lb_m)

W_H .... Heater weight (lb_m)

W_R .... Regenerator weight (lb_m)

W_s .... Heat exchanger shell weight (lb_m)

W_t .... Heat exchanger tube weight (lb_m)

ρ .... Fluid density (lb_m/ft^3)

ρ_s .... Heat exchanger shell material density (lb_m/ft^3)

ρ_t .... Heat exchanger tube material density (lb_m/ft^3)

λ .... LaGrange parameter

σ_s .... Heat exchanger shell yield strength (psi)

σ_t .... Heat exchanger tube yield strength (psi)

μ .... Fluid viscosity (lb_m/ft-sec)

k .... Fluid thermal conductivity (Btu/ft-sec-^{o}R)

Constants:

K_1 .... Defined by (1)

K_2 .... Defined by (3)

K_3 .... Defined by (4), (14) and (15)

K_4 .... Defined by (5), (14) and (15)

K_5 .... Defined by (8)

K_6 .... Defined by (16)

K_7 .... Defined by (18)

K_8 .... Defined by (20)

K_9 .... Defined by (25)

K_C .... Defined by (20)

K_H .... Defined by (20)

K_R .... Defined by (20)

K_p .... Defined by (20)

<u>**Subscripts:**</u>

1 .... Fluid inside heat exchanger tubes

2 .... Fluid in heat exchanger shell

R1 .... Regneerator tubes

R2 .... Regenerator shell

C .... Cooler

H .... Heater

CHAPTER I

INTRODUCTION

The thermodynamic potential of the closed Brayton (gas turbine) cycle power plant has been well recognized for many years. Use of this power cycle has been proposed for stationary power generating units, especially in conjunction with high temperature, gas-cooled nuclear reactors. More relevant to this study, however, is the proposed application of the closed Brayton cycle to U. S. Navy ship propulsion, primarily with conventional heat sources. Although the closed Brayton cycle power plant has not yet seen active Navy fleet application, small reseatch units are being examined and numerous studies conducted. Also, open Brayton cycle power plants have been accepted and applied on Navy destroyers, most notably the DD-963 and FFG-7 class ships. The persuit for more fuel efficient power units will enhance the opportunity for application of the closed cycle to Naval propulsion, for the closed cycle offers higher efficiencies with lower weight and volume requirements. The use of inert gases in a closed-circuit system eliminates the need for large topside vents and stacks for air intake and exhaust as with the open-cycle units; thus, permitting better

utilization of arrangement space. Developments in high temperature materials for marinized turbomachinery and heat exchangers will enable realization of the important closed cycle attributes.

One of the primary goals of conceptual ship design is to develop a ship system concept with a low acquisition cost. Often in conceptual design, ship displacement is used as surrogate for cost, ie. minimize cost by minimizing the ship displacement. Use of the closed Brayton cycle propulsion system in a concept design will entail design selection of many system variables. To a great extent, the operating variables will be governed by the ship power requirements and auxilliary system constraints. One of the major cycle operating parameters is the total cycle design pressure loss. Once all of the cycle operating parameters have been established, this overall pressure loss must be allocated in order to design the heat exchanger components. The pressure drop is an important variable in determining the size and weight of a heat exchanger; however, its impact upon other system components is nil. Consequently, for given cycle operating conditions, allocation of the total cycle design pressure loss to the heat exchangers in such a manner as to minimize the weight of the total heat exchanger package, will yield the minimum power system weight.

Often in cycle design, allocation of the cycle design pressure loss to each of the system heat exchangers is done

in a relatively arbitrary manner. The first purpose of this study is to develop an optimum allocation procedure for the cycle design pressure loss, whereby the total heat exchanger package weight is minimized. Considering the weight of the other system components to be constant, this procedure will also yield the minimum system weight. Secondly, given this optimum allocation procedure and a baseline cycle, the impact of the cycle design parameters on system weight can be examined. Thereby, the designer will be able to evaluate the relative importance of each design variable on system weight, determine basic design trends and identify and assess design trade-offs.

The basic approach of the thesis is to first describe and thermodynamically analyze the closed Brayton cycle. Next, the optimum allocation procedure is derived in chapter III for a regenerative closed cycle. Lastly, the system weight impact of each of the primary cycle and heat exchanger design parameters is examined. A worked example is provided in order to illustrate the thermodynamic and optimum design pressure loss calculations.

CHAPTER II

THE CLOSED BRAYTON CYCLE

The Brayton cycle is the basic gas turbine cycle. It and the Rankine or basic steam power cycle have been the primary Naval non-nuclear power sources. The closed Brayton cycle is distinguished from these by its basic single-phase gas closed-circuit nature. The purpose of this chapter will be to define and describe the closed Brayton cycle through thermodynamic comparison with the simple steam and gas turbine cycles. Working fluids for the closed Brayton cycle will be discussed and a detailed cycle thermodynamic analysis presented.

A. Rankine Cycle

The Rankine cycle is the steam power cycle employed on most Navy ships. Figure 1 depicts the basic cycle which consists of a boiler, turbine, condenser,and pump in a closed two-phased fluid system. The pump raises water to boiler pressure at 1. Heat is added to boil the pressurized water to form saturated steam at 2. The steam is expanded through the turbine, producing power, and the turbine exhaust steam is condensed to saturated water at 4 where the cycle begins. Generally, a superheater is added to the system to obtain a

Figure 1

Rankine Cycle Schematic

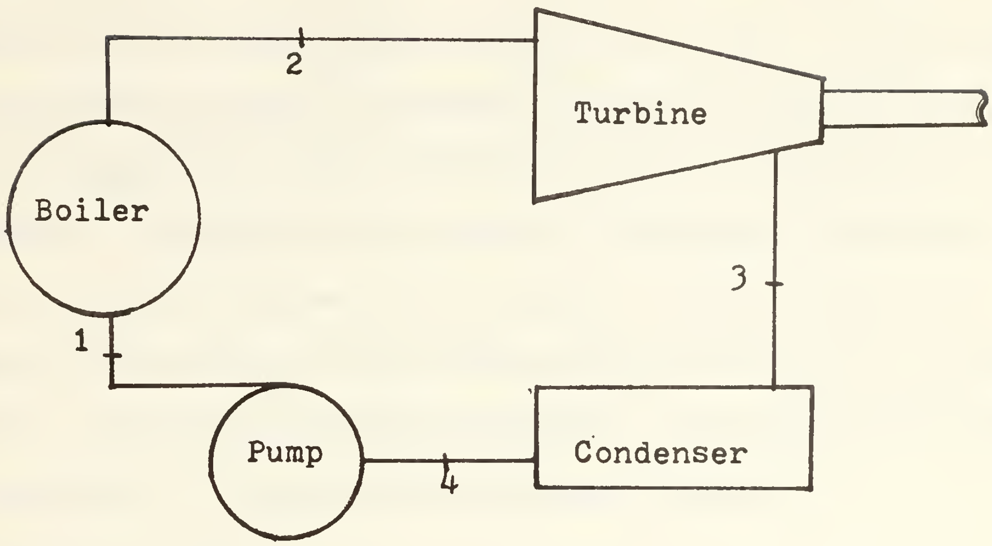
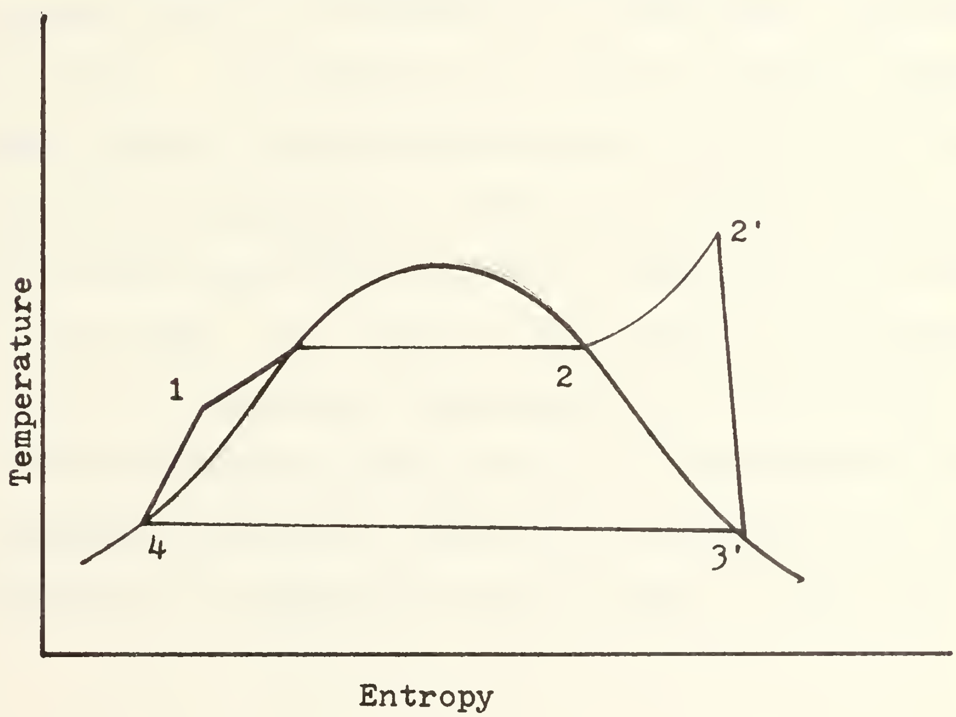


Figure 2

Rankine Cycle Temperature-Entropy Diagram



higher thermal effieiency by increasing the steam turbine inlet temperature. This is shown in the temperature-entropy diagram of figure 2. Additional heat is added to the saturated steam in the superheater from 2 to 2'. after which the superheated steam is expanded through the turbine to 3' and then condensed to 4. Lines 1-4 and 2'-3' represent non-isentropic compression and expansion because of practical efficiencies and losses in the cycle. Further gains in thermal efficiencies can be achieved by reducing the condenser pressure, raising the boiler pressure, increasing the amount of superheat or regenerative feedwater heating.

B. <u>Brayton Cycle</u>

In contrast to the Rankine cycle, the Brayton cycle is a single-phase gaseous cycle. As shown in figure 3, the simple Brayton cycle is composed of a compressor, combustor and gas turbine. Atmospheric air is drawn into the compressor, compressed, heated, expanded through the turbine and finally exhausted to the atmosphere. This is essentially the cycle employed on both the DD-963 and FFG-7 class ships now being built for the Navy. The simple closed cycle is distinguished from this by the continuous recirculation of the working fluid. This eliminates contamination of turbomachinery and heat exchanger surfaces by ingestion of foreign objects or the circulation of combustion product gases. Since the

Figure 3

Simple Open Brayton Cycle

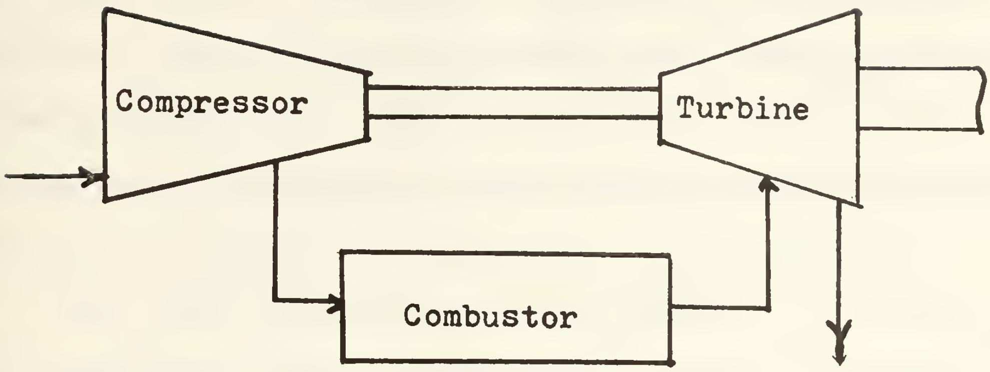
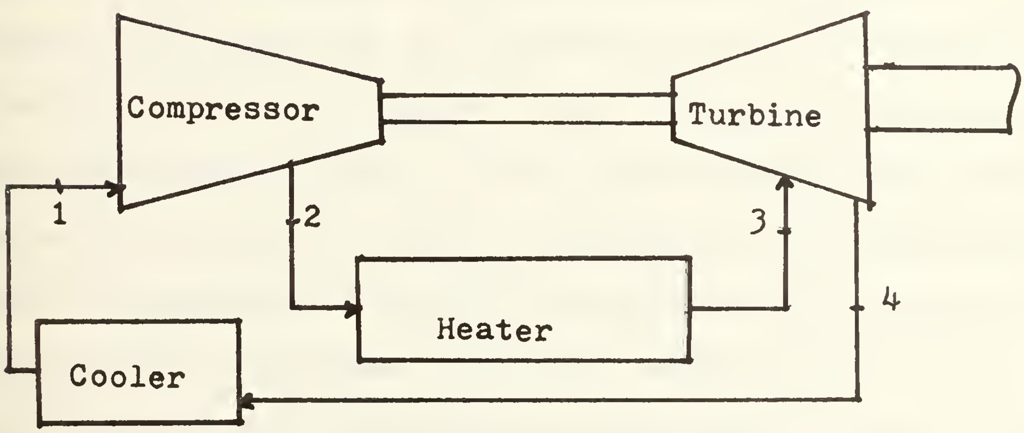


Figure 4

Simple Closed Brayton Cycle



closed cycle does not depend upon atmospheric inlet pressures and temperatures, the entire system can be pressurized. This permits high heat recovery at lower pressure ratios and higher system pressures; resulting in higher gas densities, thus smaller components, and improved gas heat transfer properties. The closed system is also more versatile in the range of heat sources which may be employed and more susceptible to automatic control systems.

The closed cycle is shown in figure 4. The flow of low temperature gas is compressed by a gas compressor to state 2. The compressed gas then passes through a heater where it is raised to a high temperature at 3. The hot pressurized gas is expanded through a gas turbine to produce sufficient power for driving the compressor and the required load. The gas is exhausted from the turbine at 4 and then cooled to a low temperature at 1 in a cooler. This is also shown on the temperature-entropy diagram of figure 5. The thermal efficiency of the closed cycle may be improved by resorting to a more complex cycle. Usually a regenerative heat exchanger is added to the cycle whereby some of the waste heat energy in the hot, low-pressure turbine exhaust gas is transfered to the cold, high-pressure gas after it has left the compressor but before entering the heater. The improvement in cycle efficiency results from the internal preheating of the working gas; thus reducing the amount of external heat energy which nust be supplied. The regenerative

cycle and its corresponding temperature-entropy diagram are shown in figure 6. Other methods of improving the cycle efficiency include use of the cycle optimum compressor pressure ratio, increased turbine inlet temperature, intercooling between low and high presure compressors and reheating between high and low pressure turbines. A detailed analysis of the closed Brayton cycle operating parameters will be presented in chapter 4.

Figure 5.

Closed Brayton Cycle Temperature-Entropy Diagram

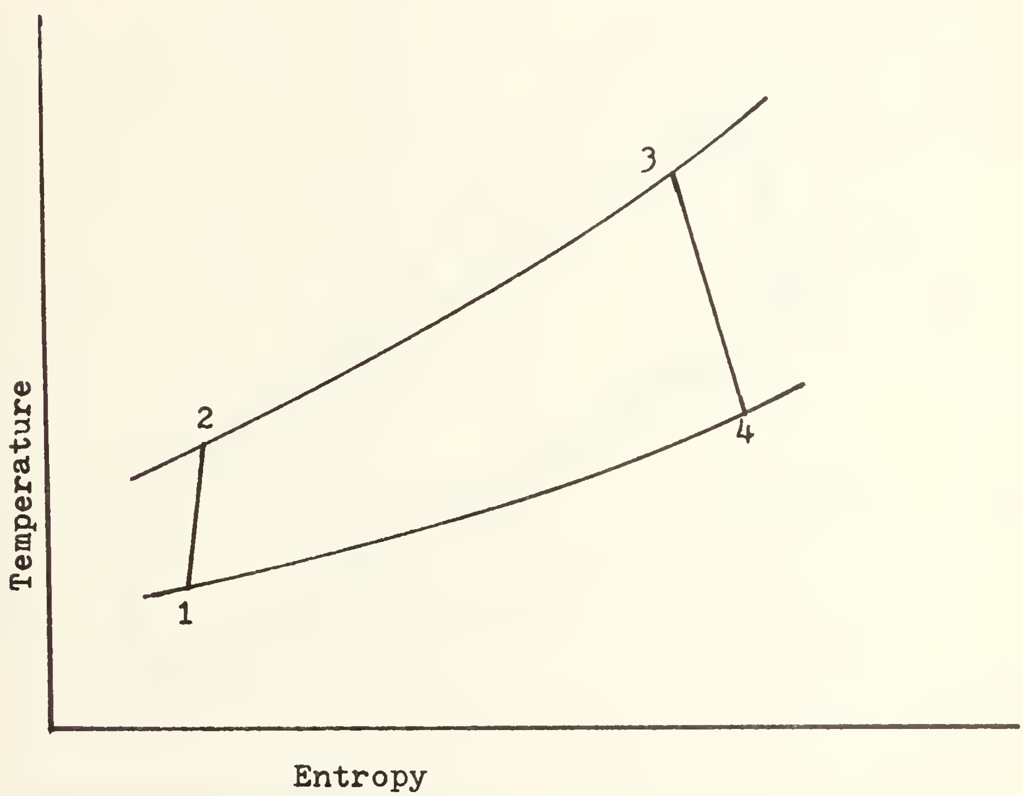
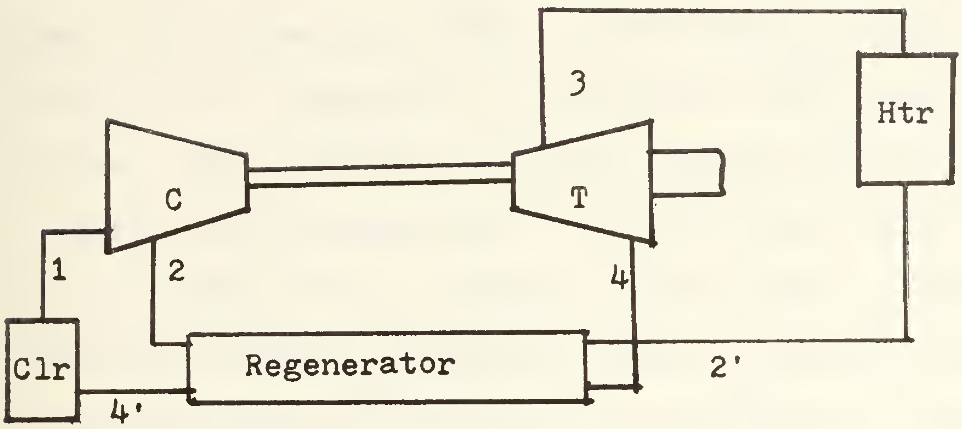
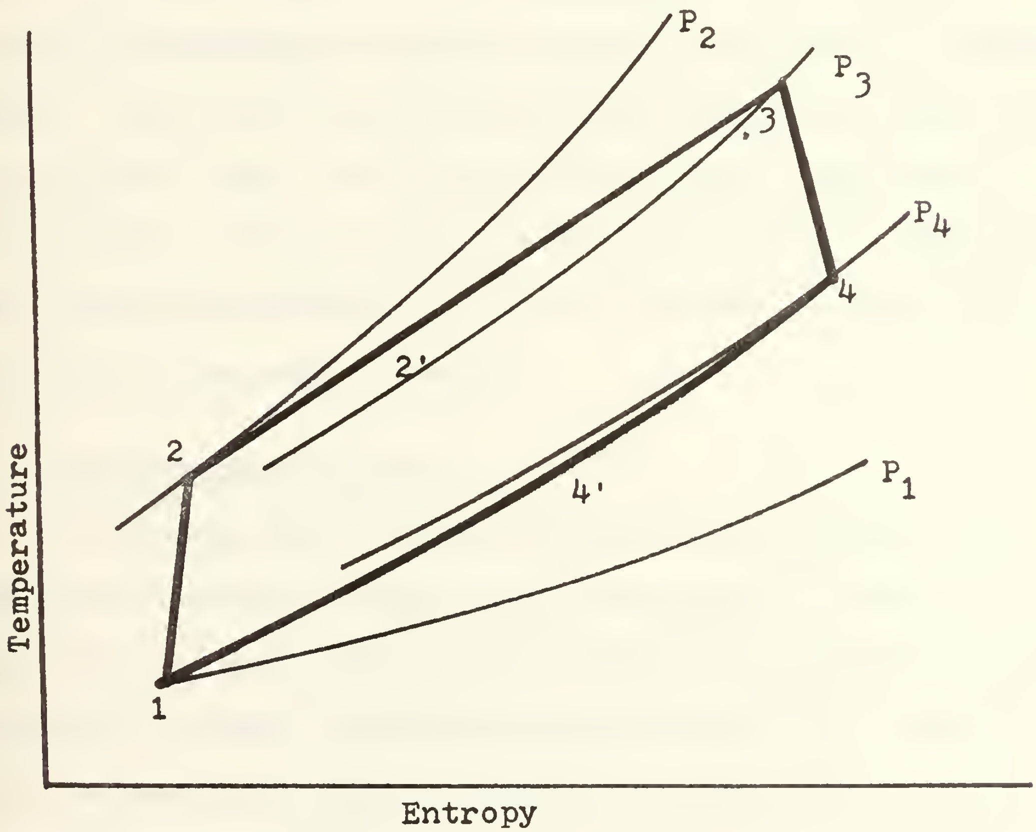


Figure 6

Regenerative Closed Brayton Cycle





C. Working Fluids

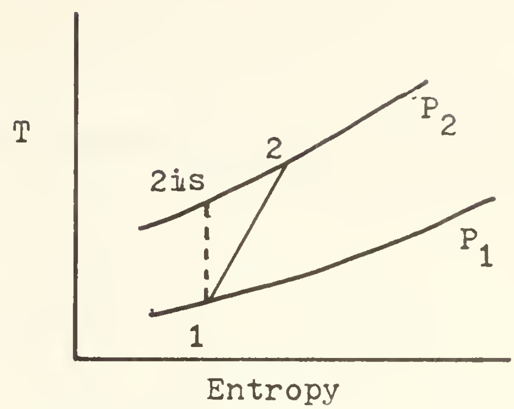
A very attractive aspect of the closed Brayton cycle lies in its flexibility in working gases. Open cycles are limited to air as the working fluid; however, since the working fluid is sealed, the closed cycle permits use of gases other than air. Many benefits accrue for such flexibility. Selection of relatively inert gases such as helium, neon, nitrogen or carbon dioxide enables use of alloys which possess beneficial high temperature properties but poor resistance to oxygen; thus higher system temperatures can be attained. Also, compressor power requirements are significant in the Brayton cycles. In order to reduce this, it is advantageous to use dense gases, for which increasing molecular weight and pressure increase density. The noble gases have higher molecular weights and high specific heat ratios which give higher aerodynamic efficiencies, improved heat transfer and smaller components. These gases are expensive, but in a compact recirculating system the inventory is small.

D. Thermodynamic Analysis

The regenerative closed Brayton cycle schematic and temperature-entropy diagram were presented in figure 6. This cycle will be thermodynamically analyzed by examining each component in order to determine each state point and the cycle thermal efficiency. An ideal working gas will be assumed throughout.

1. Compressor

Usually the compressor inlet temperature (T_1), the compressor pressure ratio (P_2/P_1) and the compressor efficiency (η_C) are specified. The compressor efficiency is defined as the ratio of the ideal compressor work to the actual compressor work.



Then, for an ideal gas where the enthalpy (H) equals the product of the specific heat and the absolute temperature,

$$\eta_c = \frac{\text{Ideal Work}}{\text{Actual Work}} = \frac{w(H_{2is} - H_1)}{w(H_2 - H_1)} = \frac{T_{2is} - T_1}{T_2 - T_1}$$

$$\eta_c = \frac{T_1\left(\frac{T_{2is}}{T_1} - 1\right)}{T_2 - T_1}$$

For an isentropic process such as from 1 to 2is,

$$\frac{T_{2is}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}$$

and therefore;

$$\eta_c = \frac{T_1\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}{T_2 - T_1}$$

Rearranging yields;

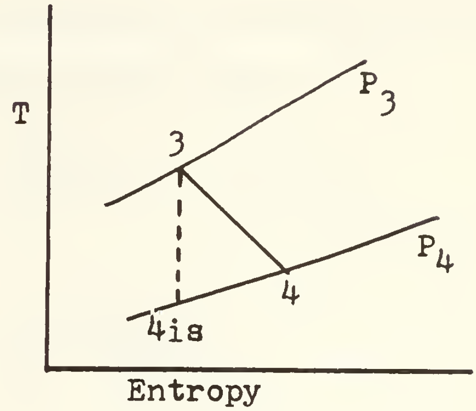
$$T_2 = T_1\left\{1 + \frac{1}{\eta_c}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]\right\}$$

and the compressor work is:

$$W_c = \frac{w C_p T_1}{\eta_c}\left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

2. Turbine

Usually the turbine inlet temperature (T_3), the turbine efficiency (η_t) and total cycle design pressure loss $(\Delta P/P)_T$ are specified. The turbine pressure ratio (P_4/P_3) can be determined from the total cycle design pressure loss, which is defined as:



$$(\Delta P/P)_T = (\Delta P/P)_{R1} + (\Delta P/P)_{R2} + (\Delta P/P)_H + (\Delta P/P)_C$$

where;

$(\Delta P/P)_{R1}$ = regenerator pressure loss (high pressure side)

$(\Delta P/P)_{R2}$ = regenerator pressure loss (low pressure side)

$(\Delta P/P)_H$ = heater pressure loss

$(\Delta P/P)_C$ = cooler pressure loss

and all pressure losses are for the working fluid.

$$\frac{P_3}{P_4} = \frac{P_1 - (\Delta P_{R1} + \Delta P_H)}{P_2 + (\Delta P_{R2} + \Delta P_c)}$$

$$\frac{P_3}{P_4} = \frac{P_2}{P_1}\,\frac{1-\left(\frac{\Delta P_{R1}}{P_2} + \frac{\Delta P_H}{P_2}\right)}{1+\left(\frac{\Delta P_{R2}}{P_1} + \frac{\Delta P_c}{P_1}\right)} \cdot \frac{1-\left(\frac{\Delta P_{R2}}{P_1} + \frac{\Delta P_c}{P_1}\right)}{1-\left(\frac{\Delta P_{R2}}{P_1} + \frac{\Delta P_{R2}}{P_1}\right)}$$

For small pressure drops, second order terms may be neglected, therefore:

$$\frac{P_3}{P_4} \cong \left[1 - \left(\frac{\Delta P_{R1}}{P_2} + \frac{\Delta P_H}{P_2} + \frac{\Delta P_{R2}}{P_1} + \frac{\Delta P_c}{P_1} \right)\right]$$

$$\frac{P_3}{P_4} = \frac{P_2}{P_1}\left(1 - \frac{\Delta P}{P}\right)$$

Then, the turbine efficiency is defined as the ratio of the actual turbine work to the ideal turbine work:

$$\eta_t = \frac{\text{Actual Work}}{\text{Ideal Work}} = \frac{H_3 - H_4}{H_3 - H_{4is}} = \frac{T_3 - T_4}{T_3 - T_{4is}}$$

$$\eta_t = \frac{T_3 - T_4}{T_3\left[1 - \frac{T_{4is}}{T_3}\right]} = \frac{T_3 - T_4}{T_3\left[1 - \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right]}$$

Rearranging yields:

$$T_4 = T_3\left\{1 - \eta_t\left[1 - \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}}\right]\right\}$$

and the turbine work is:

$$W_t = wC_pT_3\eta_t\left[1 - (P_4/P_3)^{\frac{\gamma-1}{\gamma}}\right]$$

3. Regenerator

The regenerator is a heat exchanger that uses the turbine exhaust gas to preheat the compressor exhaust gas while at the same time cooling the turbine exhaust gas. This reduces the amount of heat required from point 2 to point 3 and also reduces the cooling required from point 4 to point 1. With

the regenerator, heat input is only needed from points 2' to 3 on figure 6. Cooling is only required from points 4' to 1.

Usually the regenerator effectiveness is specified. For constant specific heats, this is defined by

$$\epsilon_R = \frac{T_{2'} - T_2}{T_4 - T_{4'}}$$

Then, $T_{2'} = T_2(1 - \epsilon_R) + \epsilon_R T_4$. A heat balance on the regenerator yields; $T_{4'} = T_4 - T_2 + T_{2'}$.

4. Heater

For this case, it is assumed that the heater is a heat exchanger in which the working fluid is heated from states 2' to 3 by hot gases from an external combustion loop. Usually the combustion loop efficiency (η_b) is specified. The heater energy input (Q_H) is

$$Q_H = wC_p(T_3 - T_{2'})$$

5. Cycle Efficiency

The cycle thermal efficiency is defined as the ratio of the net work output to the heater energy input. The cycle net work (W_{net}) is specified. From the compressor and turbine work equations and W_{net}, the cycle mass flow rate can be determined since, $W_t - W_c = W_{net}$. W_t and W_c are determined in terms of the mass flow rate (w); therefore, the mass flow rate can be calculated. From this, Q_H is determined and the cycle thermal efficiency found from

$\eta = W_{net}/Q_H$. The specific fuel consumption (sfc) is defined as the amount of fuel required per hour to produce one horsepower, ie. $sfc = w_{fuel}/W_{net}$. A heat balance on the heater yields, $Q_H = \eta_b w_{fuel} HV$, where HV is the heating value of the fuel. Also, since $Q_H = W_{net}/\eta$, then

$$sfc = 1/\eta\eta_b HV$$

Thus, all of the state points, the cycle thermal efficiency and the specific fuel consumption have been determined.

CHAPTER III

HEAT EXCHANGER WEIGHT OPTIMIZATION

For given cycle operation conditions, minimization of the closed Brayton cycle power plant weight can be achieved through minimizing the weight of the system heat exchanger package. The individual heat exchangers are designed based upon the task which each must perform as defined by the parameters mass flow rates, inlet and outlet temperatures, working fluids and pressure drop. Of these parameters only allocation of the total cycle pressure loss to each of the heat exchangers is at the discretion of the designer. It is the purpose here to develop a method by which this allocation of the overall cycle pressure loss can be accomplished in such a manner as to minimize the sum of the individual heat exchanger weights. This is done for a heater-cooler-regenerator system, all of which are single-pass, counter-flow, bare tube shell-and-tube heat exchanger. First, equations are derived for the heat exchanger length and number of tubes in terms of the pressure drop; next, heat exchanger weight estimating relationships are developed; and lastly, weight minimization equations are derived using the LaGrange multiplier technique. Throughout, the subscript 1 refers to the fluid inside the heat exchanger

tubes and the subscript 2 refers to the fluid in the shell.

A. Heater and Regenerator

For fluids in fully developed flow through tubes in the low trubulent range of Reynolds Number (Re) between 2300 and 25000, the dimensionless Moody friction factor (f) approximation is $f = 0.0791\ Re_D^{-0.25}$ where the friction factor is defined by the relation $\Delta P_f = 4f(L/D)G^2/2g_0$ and the Reynolds Number by Re=GD where $G=w/A_x$.

The overall pressure drop in a tube is the sum of entrance and exit losses, form and acceleration losses, and frictional losses. The frictional pressure drop can be calculated as

$$\Delta P_f = 0.3164 Re_D^{-0.25}(L/D)G^2/2g_0\rho$$

Substituting $G=w/A_x$, and $A_{x1}=(\pi/4)D_1^2 n$,

$$\Delta P_f = \left[\frac{0.1582\,\mu_1^{0.25}\left(\frac{4w_1}{\pi}\right)^{1.75}}{\rho_1 g_o}\right]\frac{L}{n^{1.75}D^{4.75}}$$

The entrance and exit effects are considered neglegible for L/D greater than 60, which is the case here. Form and acceleration losses are reasonably approximated as 10% of the frictional pressure loss for the range of pressures and temperatures; therefore, the pressure drop inside the tubes is expressed as

$$\Delta P_1 = \left[\frac{0.174\,\mu_1^{0.25}\left(\frac{4w_1}{\pi}\right)^{1.75}}{\rho_1 g_o}\right]\frac{L}{n^{1.75}D_1^{4.75}} \qquad (1)$$

$$\Delta P_1 = K_1 \frac{L}{n^{1.75} D_1^{4.75}}$$

(1)

The pressure drop in the shell can similarly be expressed; however, the concept of equivalent hydraulic diameter (D_2) must be used in place of D_1 and the flow area is defined as

$$A_{x2} = (\pi/4)D_0 D_2 n, \quad D_2 = qD_1, \quad D_0 = D_r D_1 \qquad (2)$$

The parameter q depends upon the tube geometry. Then for a given tube geometry;

$$\Delta P_2 = \left[\frac{0.174 \mu_2^{0.25} \left(\frac{4\omega_2}{\pi}\right)^{1.75}}{\rho_2 g_0 D_r^{1.75} q^3}\right] \frac{L}{n^{1.75} D_1^{4.75}}$$

$$\Delta P_2 = K_2 \frac{L}{n^{1.75} D_1^{4.75}}$$

(3)

For turbulent flow heat transfer in gases with Prandtl Number (Pr) in the range 0.5 to 1.0, the heat transfer coefficient can be expressed as $h = 0.022(k/D)Re^{0.8}Pr^{0.6}$. Then,

$$h_1 = \left[\frac{0.022 k_1 Pr_1^{0.6}}{\mu_1^{0.8}}\right] \frac{G_1^{0.8}}{D_1^{0.2}} = K_3 \frac{G_1^{0.8}}{D_1^{0.2}}$$

(4)

$$h_2 = \left[\frac{0.022 k_2 Pr_2^{0.6}}{\mu_2^{0.8}}\right] \frac{G_2^{0.8}}{D_2^{0.8}} = K_4 \frac{G_2^{0.8}}{D_2^{0.2}}$$

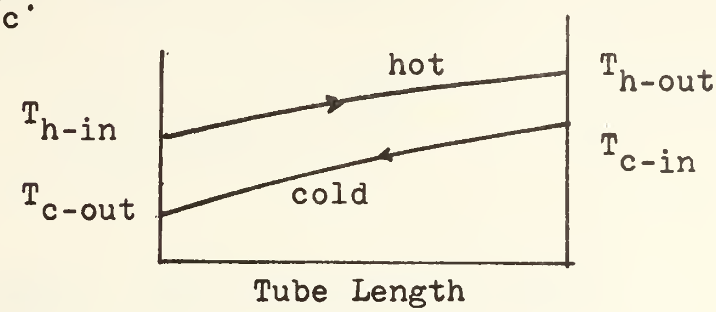
(5)

The amount of energy transferred between flowing fluids in counter-flow is expressed as $Q = UA\Delta T_{lm}$ (6)

where Q = amount of heat energy transferred between fluids
U = onerall heat transfer coefficient
A = heat transfer surface area
ΔT_{lm} = log mean temperature difference between the hot and cold fluids

The log mean temperature difference is defined as below

for $w_h \neq w_c$.



$$T_{lm} = \frac{(T_{h-out} - T_{c-in}) - (T_{h-in} - T_{c-out})}{\ln\left(\dfrac{T_{h-out} - T_{c-in}}{T_{h-in} - T_{c-out}}\right)} \qquad (6a)$$

When $w_h = w_c$ as in a regenerator, $T_{lm} = T_{h-out} - T_{c-in}$ (6b)

The heat transfer area is defined based upon the inside

diameter $A_1 = \pi n D_1 L$ (7)

The overall heat transfer coefficient is defined based upon the inside diameter neglecting the tube wall thermal resistance which is neglegible in the cases being studied.

$$\frac{1}{U} = \frac{1}{h_1} + \frac{D_1}{D_0}\left(\frac{1}{h_2}\right) = \frac{1}{h_1}\left[1 + \frac{D_1 h_1}{D_0 h_2}\right]$$

From equations (4) and (5),

$$\frac{h_1}{h_2} = \frac{K_3}{K_4}\left(\frac{G_1}{G_2}\right)^{0.8}\left(\frac{D_2}{D_1}\right)^{0.2}$$

From the definitions of G, A_{x1} and A_{x2} and (2);

$$\frac{h_1}{h_2} = \frac{K_3}{K_4}\left(\frac{\omega_1 D_r}{\omega_2}\right)^{0.8} g$$

$$\frac{1}{u} = \frac{D_1^{1.8}}{K_3}\left(\frac{\pi n}{4\omega_1}\right)\left[1 + \frac{K_3}{K_4 D_r}\left(\frac{w_1 D_r}{w_2}\right)^{0.8} g\right]$$

$$\frac{1}{u} = K_5 D_1^{1.8} n^{0.8} \qquad (8)$$

Substituting (7) and (8) into (6) yields;

$$Q = \frac{n\pi D_1 L \Delta T_{em}}{K_4 D_1^{1.8} n^{0.8}} = \left(\frac{\pi \Delta T_{em}}{K_4}\right)\frac{L n^{0.2}}{D_1^{0.8}}$$

Now, the amount of heat energy transferred between fluids is a known quantity since;

$$Q = wC_p(T_{out}-T_{in}) \qquad (9)$$

Therefore;

$$L n^{0.2} = \left(\frac{K_4 Q}{\pi \Delta T_{em}}\right) D_1^{0.8} \qquad (10)$$

From (1);

$$L n^{-1.75} = \frac{\Delta P_1}{K_1} D_1^{4.75} \qquad (11)$$

Solving (10) and (11) simultaneously yields:

$$n = \left(\frac{K_1 K_4 Q}{\pi \Delta T_{em}}\right)^{0.513} \Delta P^{-0.513} D_1^{-2.026} \qquad (12)$$

$$L = \left(\frac{K_4 Q}{\pi \Delta T_{lm}}\right)^{0.897} \left(\frac{\Delta P_1}{K_1}\right)^{0.103} D_1^{1.205}$$

(13)

B. Cooler

Because the cooler is a liquid-gas heat exchanger, some minor changes must be made to the above derivations. For turbulent flow heat transfer in a liquid, the heat transfer coefficient can be expressed using the Dittus-Boelter equation $h = 0.023(k/D)Re^{0.8}Pr^{0.4}$.

If it is assumed that the cooling water flows through the tubes and the helium in the shell;

$$h_1 = \left[\frac{0.023\, k_1 Pr_1^{0.4}}{\mu_1^{0.8}}\right] \frac{G_1^{0.8}}{D_1^{0.2}} = K_3 \frac{G_1^{0.8}}{D_1^{0.2}}$$

$$h_2 = \left[\frac{0.022\, k_2 Pr_2^{0.6}}{\mu_2^{0.8}}\right] \frac{G_2^{0.8}}{D_2^{0.2}} = K_4 \frac{G_2^{0.8}}{D_2^{0.2}}$$

(14)

Likewise, if the helium flows in the tubes and cooling water in the shell;

$$h_1 = \left[\frac{0.022\, k_1 Pr_1^{0.6}}{\mu_1^{0.8}}\right] \frac{G_1^{0.8}}{D_1^{0.2}} = K_3 \frac{G_1^{0.8}}{D_1^{0.2}}$$

$$h_2 = \left[\frac{0.023\, k_2 Pr_2^{0.4}}{\mu_2^{0.8}}\right] \frac{G_2^{0.8}}{D_2^{0.2}} = K_4 \frac{G_2^{0.8}}{D_2^{0.2}}$$

(15)

All other constants will remain the same as for the case of the heater and regenerator.

C. <u>Heat Exchanger Weight</u>

The heat exchangers are composed of three major pieces. These are the tube nest, the shell and the end pieces and miscellaneous structure. The weight of the tube nest can be expressed as;

$$W_t = \frac{\pi \rho_t}{4}\left(D_r^2 - 1\right) L n D_1^2 = K_6 L n D_1^2 \tag{16}$$

where ρ_t is the tube material density.

The weight of the heat exchanger shell can be approximated using the hoop stress formula to determine the required shell thickness.

$$t_s = \frac{P^* D_s}{2\sigma_s}$$

where P^* = the design pressure = 1.5 P_{shell}

σ_s = the shell material yield strength

The diameter of the shell (D_s) can be found as follows;

$$A_{xT} = \frac{\pi}{4} D_s^2 = A_{x1} + A_{x2} = \frac{\pi}{4}\left(D_1^2 n + D_o D_2 n\right)$$

$$D_s = D_1 \left[n\left(1 + D_r g\right)\right]^{0.5} \tag{17}$$

Then the weight of the shell for t_s much less than D_s is;

$$W_s = \rho_s L \pi D_s t_s = \frac{\rho_s L \pi P^* D_s^2}{2\sigma_s}$$

$$W_s = \left[\frac{\pi \rho_s P^* (1 + D_r g)}{2\sigma_s}\right] L n D_1^2 = K_7 L n D_1^2 \tag{18}$$

The weight of the end pieces and miscellaneous structures depends upon numerous geometrical assumption. Detailed calculations under varying assumptions shows that reasonable approximation to this weight is 10% of the total heat exchanger weight. Then,

$$W_T = 1.1(K_6 + K_7)LnD_1^2 K_p^{-0.41} \tag{19}$$

From (12) and (13),

$$W_i = 1.1(K_6+K_7)\left(\frac{K_4 Q}{\pi \Delta T_{lm}}\right)^{1.41} K_1^{0.41} D_1^{1.18} \Delta P_1^{-0.41} K_p^{-0.41}$$

$$W_i = K_i \Delta P^{-0.41} \tag{20}$$

where $i = \begin{cases} R \text{ for regenerator} \\ H \text{ for heater} \\ C \text{ for cooler} \end{cases}$ and K_p is a constant to determine the helium pressure drop (ΔP_{He}) from the fluid pressure drop inside the tubes. From (1) and (3),

$$(\Delta P_2/\Delta P_1) = K_2/K_1 = K_8$$

and $K_p = \begin{cases} 1 \text{ if helium is inside the tubes} \\ K_1/K_2 \text{ if helium is in the shell} \end{cases}$

thus, $W_R = K_R \Delta P_{1R}^{-0.41}$, $W_C = K_C \Delta P_{1C}^{-0.41}$, and $W_H = K_H \Delta P_{1H}^{-0.41}$

and $W = W_R + W_C + W_H$.

D. Weight Optimization

The LaGrange multiplier technique can now be applied to minimize the heat exchanger package weight for the given closed Brayton cycle, subject to a known constraint upon the total cycle pressure loss $(\Delta P/P)_T$.

$$(\Delta P/P)_T = (\Delta P/P_H)_{R1} + (\Delta P/P_L)_{R2} + (\Delta P/P_H)_H + (\Delta P/P_L)_C \quad (21)$$

where $(\Delta P/P)_T$ is the total cycle design pressure loss

$(\Delta P/P_H)_{R1}$ is the regenerator tube pressure loss

$(\Delta P/P_L)_{R2}$ is the regenerator shell pressure loss

$(\Delta P/P_H)_H$ is the heater pressure loss

$(\Delta P/P_L)_C$ is the cooler pressure loss

P_H is the compressor outlet (high) pressure

P_L is the compressor inlet (low) pressure

and all pressure drops are for the helium. Therefore, the problem can be stated as:

$$\text{Min} \quad W = W_R + W_H + W_C$$

Subject to a known $(\Delta P/P)_T$

The solution by the LaGrange multiplier technique follows:

$$W = K_R P_H^{-0.41} \left(\frac{\Delta P}{P_H}\right)_{R1}^{-0.41} + K_H P_H^{-0.41} \left(\frac{\Delta P}{P_H}\right)_H^{-0.41} + K_C P_L^{-0.41} \left(\frac{\Delta P}{P_L}\right)_C^{-0.41} \quad (22)$$

$$\left(\frac{\Delta P}{P}\right)_T = \left(1 + \frac{P_H K_{8R}}{P_L}\right)\left(\frac{\Delta P}{P_H}\right)_{R1} + \left(\frac{\Delta P}{P_H}\right)_H + \left(\frac{\Delta P}{P_L}\right)_C \quad (23)$$

In general,

$$\frac{\partial W}{\partial\left(\frac{\Delta P}{P}\right)_i} = \lambda \frac{\partial\left(\frac{\Delta P}{P}\right)_T}{\partial\left(\frac{\Delta P}{P}\right)_i}$$

Taking partial derivatives yields,

$$-0.41\, K_R\, P_H^{-0.41} \left(\frac{\Delta P}{P_H}\right)_{R1}^{-1.41} = \lambda\left(1 + \frac{P_H K_{gR}}{P_L}\right)$$

$$-0.41\, K_H\, P_H^{-0.41} \left(\frac{\Delta P}{P_H}\right)_H^{-1.41} = \lambda \qquad (24)$$

$$-0.41\, K_c\, P_L^{-0.41} \left(\frac{\Delta P}{P_L}\right)_c^{-1.41} = \lambda$$

Solving for $(\Delta P/P_H)_{R1}$ and substituting into (23) gives the relationships for the "optimum" heat exchanger pressure losses.

$$\left(\frac{\Delta P}{P}\right)_T = \left\{1 + \frac{P_H K_{gR}}{P_L} + \left[\frac{K_H}{K_R}\left(1 + \frac{P_H K_{gR}}{P_L}\right)\right]^{0.71} + \left[\frac{K_c}{K_R}\left(1 + \frac{P_H K_{gR}}{P_L}\right)\left(\frac{P_H}{P_L}\right)^{0.41}\right]^{0.71}\right\}\left(\frac{\Delta P}{P_H}\right)_{R1}$$

$$\left(\frac{\Delta P}{P}\right)_T = K_q \left(\frac{\Delta P}{P_H}\right)_{R1}$$

$$\left(\frac{\Delta P}{P_H}\right)_{R1}^* = \left(\frac{1}{K_q}\right)\left(\frac{\Delta P}{P}\right)_T$$

$$\left(\frac{\Delta P}{P_L}\right)_{R2}^* = \frac{P_H}{P_L} K_{gR} \left(\frac{\Delta P}{P_H}\right)_{R1}^* \qquad (25)$$

$$\left(\frac{\Delta P}{P_H}\right)_H^* = \left[\frac{K_H}{K_R}\left(1 + \frac{P_H K_{gR}}{P_L}\right)\right]^{0.71}\left(\frac{\Delta P}{P_H}\right)_{R1}^*$$

$$\left(\frac{\Delta P}{P_L}\right)_c^* = \left[\frac{K_c}{K_R}\left(1 + \frac{P_H K_{gR}}{P_L}\right)\left(\frac{P_H}{P_L}\right)^{0.41}\right]^{0.71}\left(\frac{\Delta P}{P_H}\right)_{R1}^*$$

Substitution of the optimum heat exchanger design pressure losses from (25) into (12), (13), (17), (19) and (22) will yield the length, number of tubes, shell diameter and weight for each heat exchanger and the total heat exchanger package weight. A worked example is provided in the appendix in order to illustrate the procedure for thermodynamic and optimum pressure loss allocation calculations.

CHAPTER IV

CYCLE DESIGN ANALYSIS

The derivation of the predeeding chapter has been based upon known heat exchanger size and cycle operating parameters. In designing the overall power cycle, the designer may have limited flexibility in determining these parameters for a given cycle configuration and power level. As with the allocation of total cycle design pressure loss addressed in the last chapter, this flexibility can be directed towards minimizing the power plant weight within certain limitations. Again assuming that, over a limited range of cycle operating parameters, the turbomachinery weight change is neglegible in comparison with that of the heat exchanger package, the influence of a limited variation in parameters on the system can be assessed by their influence upon the optimum heat exchanger weight and the cycle thermal efficiency. Through such a parametric analysis, the designer can identify basic trends and relative importance of the influence of each parameter, and overall trade-offs between decreased weight and increased efficiency.

First, the basic heat exchanger size parameters will be examined followed by an examination of the impact of the

cycle operating parameters on the cycle weight and efficiency.

A. Heat Exchanger Size Parameters

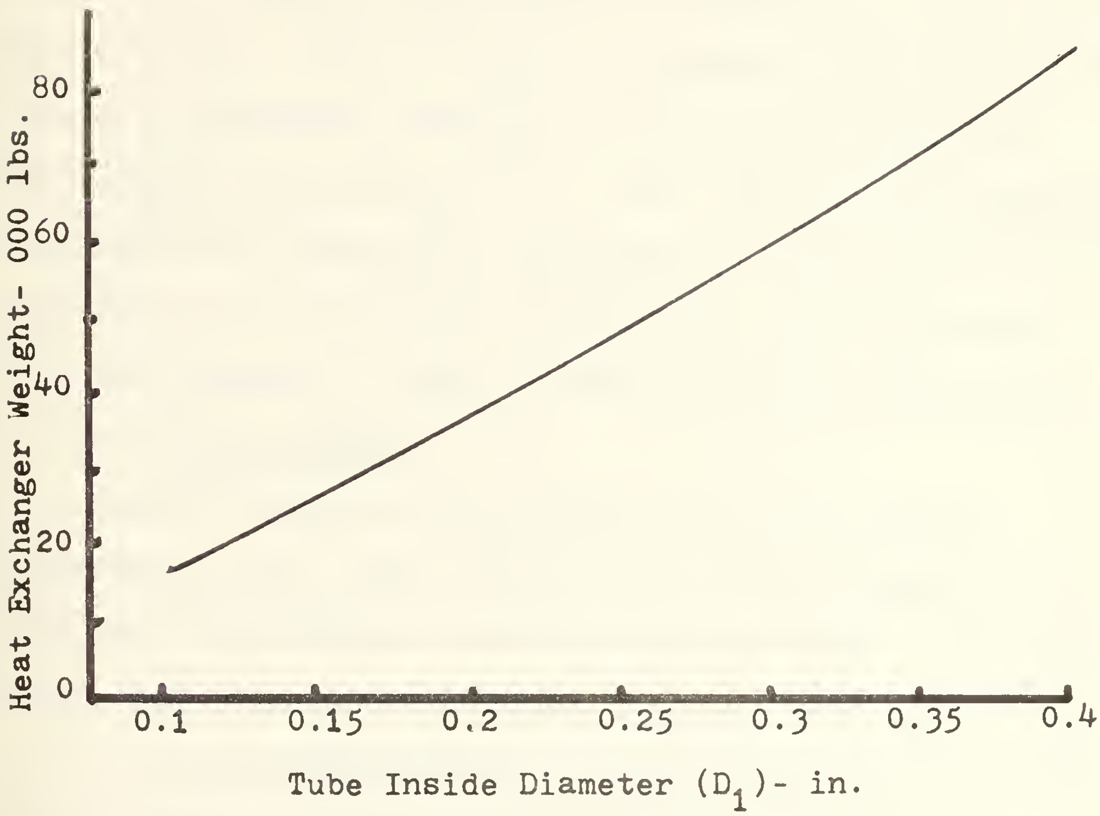
The heat exchanger size equations of the last chapter are based upon specified values for the tube size and geometry of each heat exchanger. Tube inside diameter (D_1), diameter ratio (D_r), spacing and arrangement are the primary design variables which are chosen by the designer. Prudent selection of these design variables can further reduce the heat exchanger weights. Following is a review of the impact of each design variable on heat exchanger weight.

1. Tube inside diameter (D_1).

Equation (20) and figure 7 show that the heat exchanger weight is nearly proportional to tube diameter (actually $D_1^{1.18}$); therefore the smaller practical tube diameter should be used. Aside from the basic manufactureability constraints, a very significant consideration in the heat exchangers will be the Reynolds Number (Re) of the fluids. The Re must be greater than about 2500 in order to ensure turbulent flow, which affords much better heat transfer and without which the preceeding derivations are invalid. This is likely to be the primary constraint upon tube diameter for a cooler with the cooling water flowing through the tubes. Another heat transfer consideration is maximum temperature of the tube wall. This may dictate a larger tube size in the case of heaters.

Figure 7

Influence of Tube Diameter on Heat Exchanger Weight



2. <u>Tube diameter ratio (D_r).</u>

The tube diameter ratio (D_r) expresses the relative tube wall thickness. Figure 8 shows a nearly linear relationship between heat exchanger weight and D_r. The relative impact of D_r, however, is much greater than that of tube diameter. For example, a 30% increase in D_r results in a more than 450% increase in heat exchanger weight compared to less than 20% weight increase for a 30% rise in tube diameter. This indicates a preference for the smallest tube wall thickness consistent with manufactureability and strength requirements. The tube wall maximum temperature may become the overriding constraint in the case of heaters, requiring a wall thickness greater than would otherwise have been chosen in order to reduce the wall temperature.

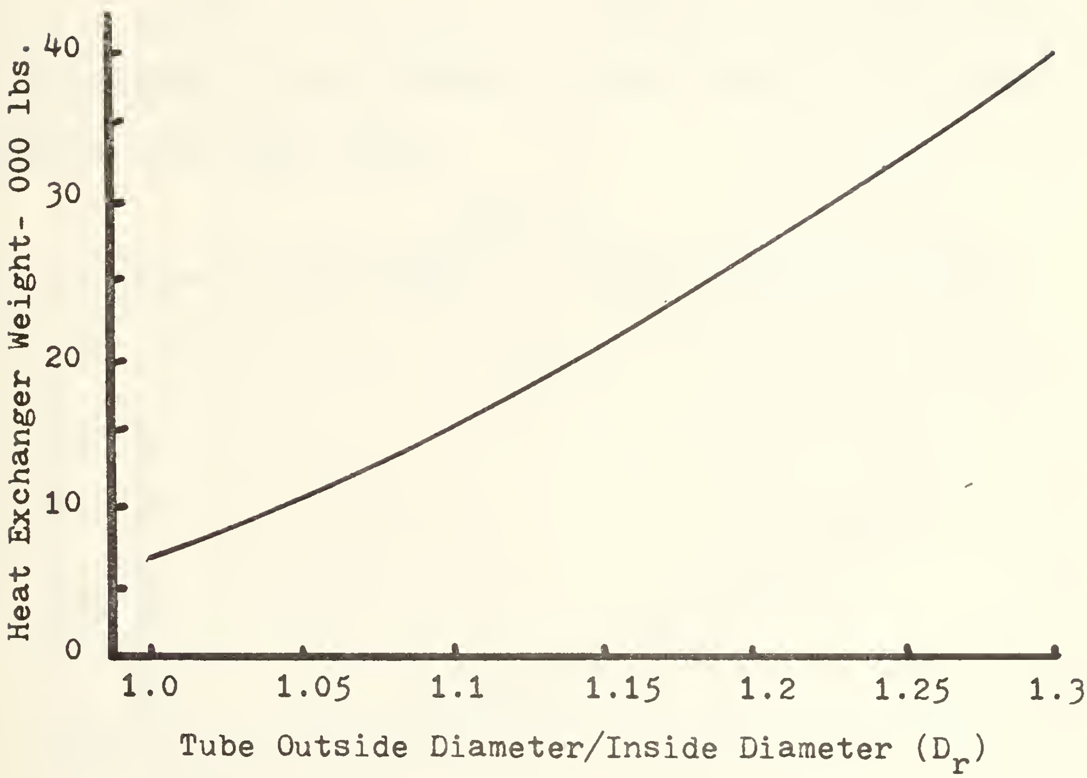
3. <u>Tube spacing.</u>

Figure 9 shows that increasing the tube spacing, expressed as s/D_o, will increase the heat exchanger weight; therefore, the smallest spacing would be sought. Reynolds Number must also be high enough to ensure turbulent flow in the heat exchanger shell. In the case of a cooler with cooling water flowing through the tubes, tube spacing becomes important to the Reynolds Number and fluid velocity inside the tubes. This relationship is expressed through the pressure drops:

$$\frac{\Delta P_1}{\Delta P_2} = \left(\frac{\mu_1}{\mu_2}\right)^{0.25} \left(\frac{\omega_1}{\omega_2}\right)^{1.75} \left(\frac{\rho_2}{\rho_1}\right) D_r^{1.75} \, q^3$$

Figure 8

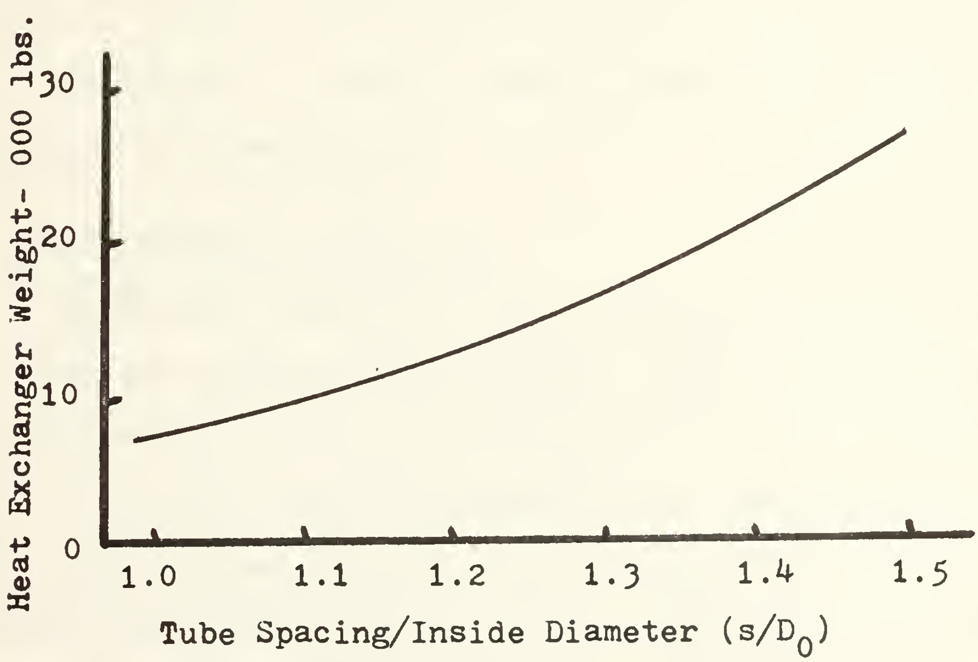
Influence of Tube Diameter Ratio on
Heat Exchanger Weight



For a given cycle, only D_r and q are left for the designer's choice. In salt water coolers, it is necessary for the water velocity to be great enough to prevent marine growth fouling on the tube walls. In order to increase ΔP_1, with a working fluid pressure drop (ΔP_2) fixed at the optimum value, the designer can increase the values of D_r or q. Since the tube spacing (s/D_0) is proportional to q through the equivalent hydraulic diameter, and because the weight effect of increasing q is less than that for D_r, one could achieve a greater $\Delta P_1/\Delta P_2$ by increasing the tube spacing. This consideration may dictate a larger spacing than might otherwise have been chosen.

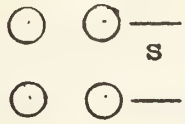
Figure 9.

Influence of Tube Spacing on Heat Exchanger Weight

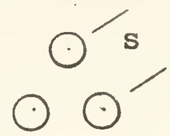


4. <u>Tube arrangement</u>.

Two widely used tube arrangements are the square pitch and equilateral triangular pitch shown below.



Square pitch



Triangular pitch

Through the concept of equivalent hydraulic diameter defined as:

$$D_2 = \frac{4\ (\text{flow area})}{\text{Wetted Perimeter}}$$

relationships can be developed for the equivalent diameter for these two geometries. They are

$$q = D_2/D_0 = 1.103(s/D_0)^2 - 1 \quad \text{(triangular pitch)}$$

$$q = D_2/D_0 = 1.271(s/D_0)^2 - 1 \quad \text{(square pitch)}$$

Therefore, for a given s/D_0, q for a square pitch will always be greater than for a triangular pitch arrangement. Then, from figure 9, one can conclude that the triangular arrangement will weigh less.

B. <u>Cycle Operating Parameters</u>

Following are analyses of the variation of cycle thermal efficiency and optimum heat exchanger weight, as calculated using the method of the previous chapter, due to limited variation in cycle operating parameters. Compressor pressure ratio, cycle maximum pressure level, regenerator effectiveness, turbomachinery inlet temperatures, turbomachinery efficiencies and total cycle design pressure loss will be examined with

respect to a baseline cycle.

The baseline cycle employed for the parametric analyses will be the 40,000 HP regenerative closed Brayton cycle with helium as working fluid. Following are the basic operating parameters for the baseline cycle.

Compressor pressure ratio	2.30
Maximum pressure level	400 psi
Regenerator effectiveness	0.88
Compressor inlet temperature	540°R
Turbine inlet temperature	1960°R
Turbine efficiency	0.91
Compressor efficiency	0.87
Total cycle pressure loss	5%
Optimum heat exchanger package weight .	28,579 lb.
Cycle thermal efficiency	41.2%
Cycle specific fuel consumption	0.372 lb_m/shp-hr

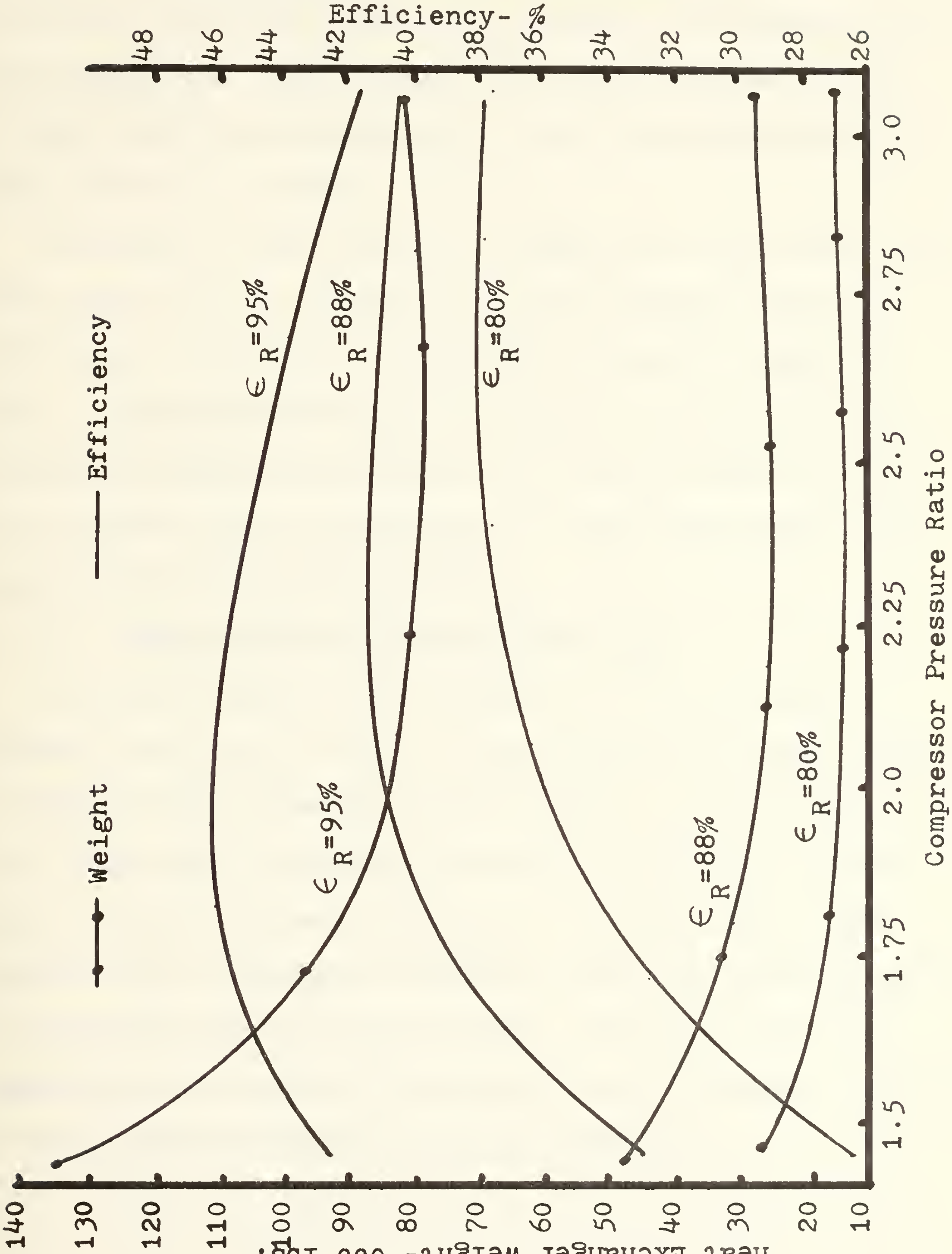
These as well as the heat exchanger geometry and tube sizes are the same as in the worked example of the appendix.

1. Compressor Pressure Ratio (CPR).

Figure 10 depicts the influence of the compressor pressure ratio on optimum heat exchanger package weight and cycle thermal efficiency for three levels of regenerator effectiveness. Both weight and efficiency curves reach extreme points with increasing CPR; However, for a given regenerator effectiveness, the minimum weight and maximum efficiency do not occur at the same CPR. For the range of effectiveness shown, the maximum efficiency occurs at a lower CPR than the minimum weight, and this CPR difference

Figure 10

Influence of Compressor Pressure Ratio on
Optimum Heat Exchanger Package Weight
and Cycle Thermal Efficiency



decreases with decreasing regenerator effectiveness. Both weight and efficiency curves are flatter at compressor pressure ratios greater than their individual optima; therefore, a smaller penalty is incurred for operating at a CPR greater than optima rather than one less than optima. At very low effectiveness levels, both curves essentially level-off at the optima.

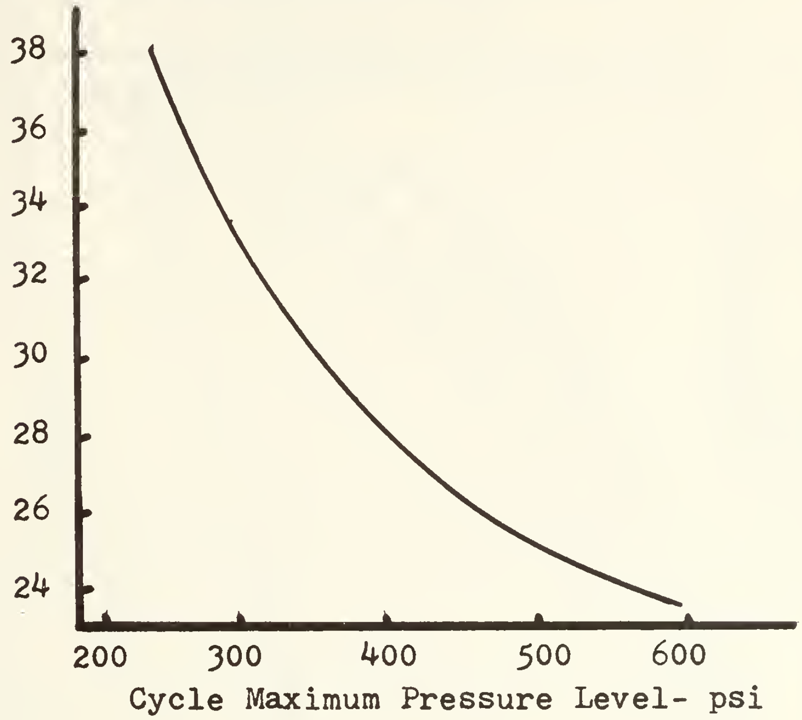
Because of this behavior of weight and efficiency with CPR, there will be a design trade-off between weight and efficiency to determine the optimum compressor pressure ratio. This trade-off will occur only over the CPR range between the maximum efficiency and minimum weight points. The trade-off will be determined by the mission endurance requirements for the ship design.

2. Cycle Maximum Pressure Level (P_H).

Figure 11 depicts the influence of cycle maximum pressure level (P_H), which is the compressor outlet pressure, on optimum heat exchanger package weight. P_H is the only one of the major operating parameters which has no impact upon the cycle thermal efficiency. As seen, the weight decreases for increasing pressure, resulting primarily from an increased working fluid density. Because there is no degradation of thermal efficiency, one would seek to operate at the highest practical operating pressure.

Figure 11

Influence of Cycle Maximum Pressure Level on Optimum Heat Exchanger Package Weight and Cycle Thermal Efficiency

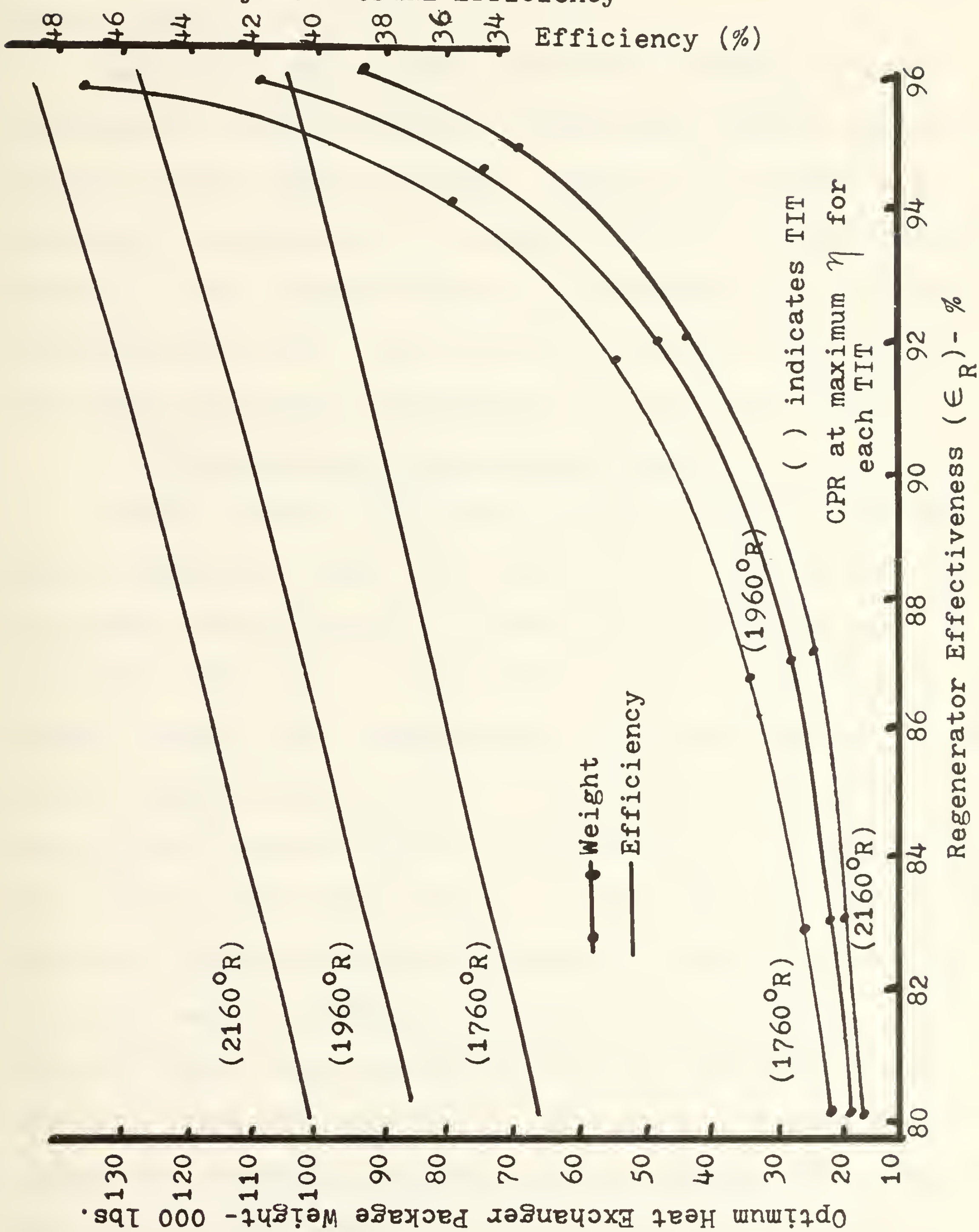


3. Regenerator Effectiveness (ϵ_R).

Figure 12 depicts the influence of regenerator effectiveness upon weight and cycle efficiency for three levels of turbine inlet temperature. Both weight and efficiency are seen to increase with increasing regenerator effectiveness. The rate of increase of efficiency appears nearly constant while that for weight is rapidly changing, especilly at high effectiveness levels. The weight penalty for increased efficiency at high effectiveness levels is quite severe--for example, not shown on the figure is the increase from 96% to 98% regenerator effectiveness which

Figure 12

Influence of Regenerator Effectiveness on
Optimum Heat Exchanger Package Weight
and Cycle Thermal Efficiency



results in a 2.5% increase in efficiency at a weight increase of 150% (over 200,000 lbs). The increase itself is over six times the total weight heat exchanger at 88% effectiveness.

Increased turbine inlet temperature appears to only increase the level of weight or efficiency without apparent effect on their rate of change. Because both weight and efficiency increase with increased regenerator effectiveness, there will be a design trade-off in determining an "optimum" effectiveness level. This trade-off will be determined by the mission endurance requirements for the ship design.

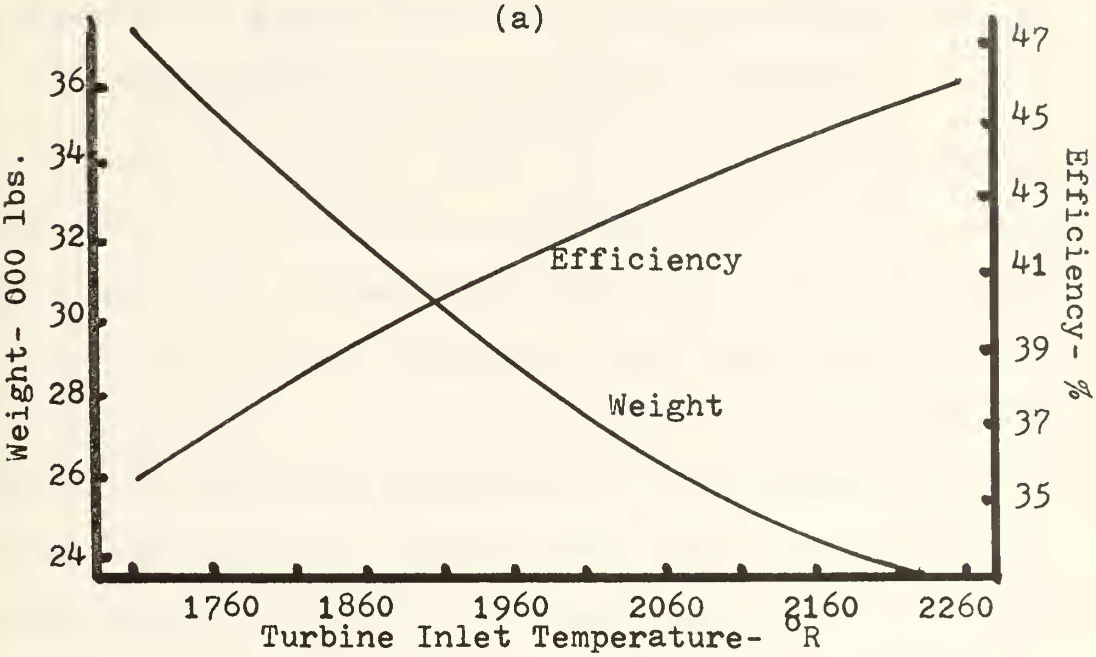
4. Turbomachinery Inlet Temperatures.

Figures 13a and 13b depict the influence of compressor inlet temperature (CIT) and turbine inlet temperature (TIT) on weight and efficiency. Because both attributes of increased efficiency and decreased weight are enhanced by higher turbine inlet temperatures, one would want to operate at the highest practical TIT. This level is generally determined by metallurgical considerations. On figure 13b, two weight curves are plotted--one for a CPR of 2.3 and the other for CPR at maximum efficiency for each CIT. Evident is a wide weight difference at higher temperatures. Only one efficiency curve is plotted because efficiency curves for both cases are essentially coincident. Because both weight and efficiency decrease with increasing CIT, there will be a design trade-off to determine the optimum CIT.

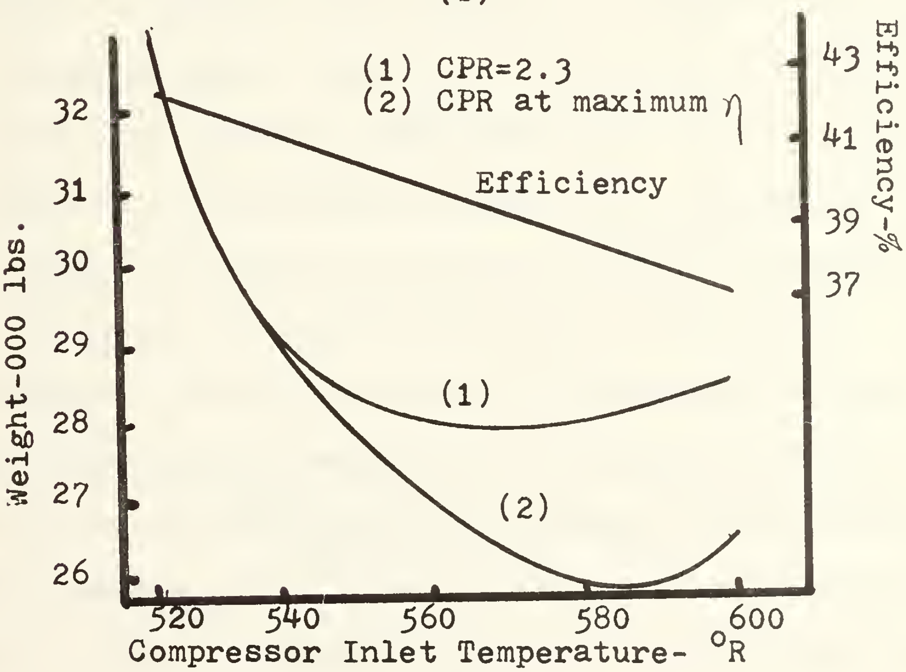
Figure 13

Influence of Turbomachinery Inlet Temperatures on Optimum Heat Exchanger Package Weight and Cycle Thermal Efficiency

(a)



(b)



This will be determined to the largest extent by the cooling water temperature: that is, the lower the cooling water temperature, the more effective the cooler. In this regard, the results are highly dependent upon the assumed values.

5. Turbomachinery Efficiencies (η_c and η_t).

Figures 14a and 14b depict the influences of compressor efficiency (η_c) and turbine efficiency (η_t) upon "optimum" weight and cycle thermal efficiency (η). Increasing either η_c or η_t will yield increasing cycle efficiency and decreasing weight. Also, there appears to be no difference between the relative influences of the compressor and turbine efficiencies. Because both yield improved weight and efficiency attributes, one would seek to use the highest practical level of turbomachinery efficiencies.

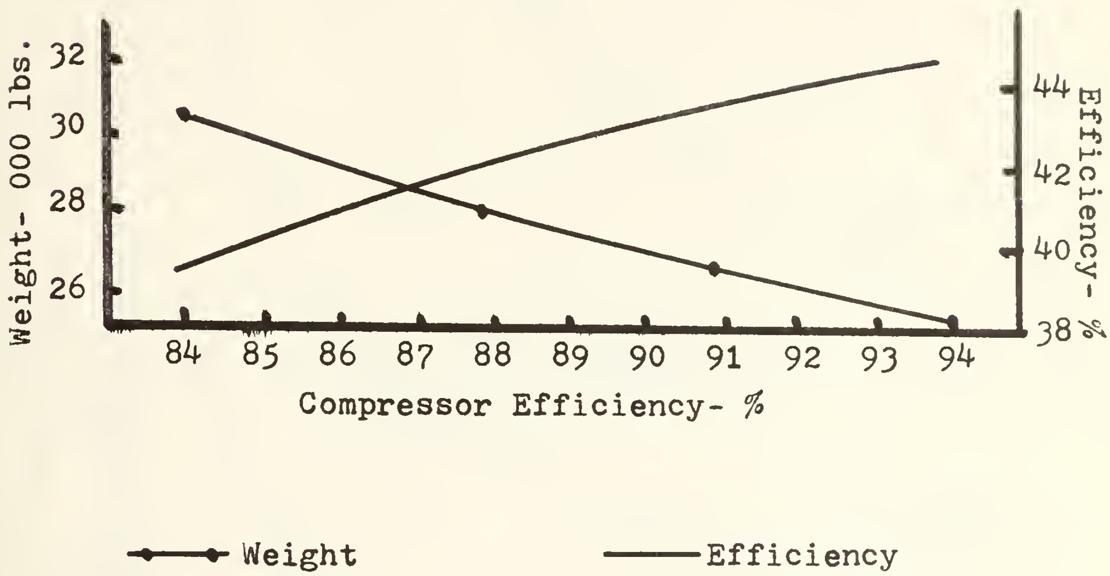
6. Total Cycle Design Pressure Loss ($\Delta P/P$).

Figure 15 depicts the influence of varying the total cycle design pressure loss ($\Delta P/P$) on optimum heat exchanger weight and cycle thermal efficiency (η). It is seen that both weight and efficiency decrease with increasing ($\Delta P/P$). Additionally, the rate of decrease in weight decreases with both increasing ($\Delta P/P$) and decreasing regenerator effectiveness. This is especially pronounced at high levels of regenerator effectiveness and low levels of ($\Delta P/P$). There will be a design trade-off between the attributes of decreased weight and increased efficiency in determining an "optimum" ($\Delta P/P$) because both weight and efficiency are

Figure 14

Influence of Turbomachinery Efficiencies on Optimum Heat Exchanger Package Weight and Cycle Thermal Efficiency

(a)



(b)

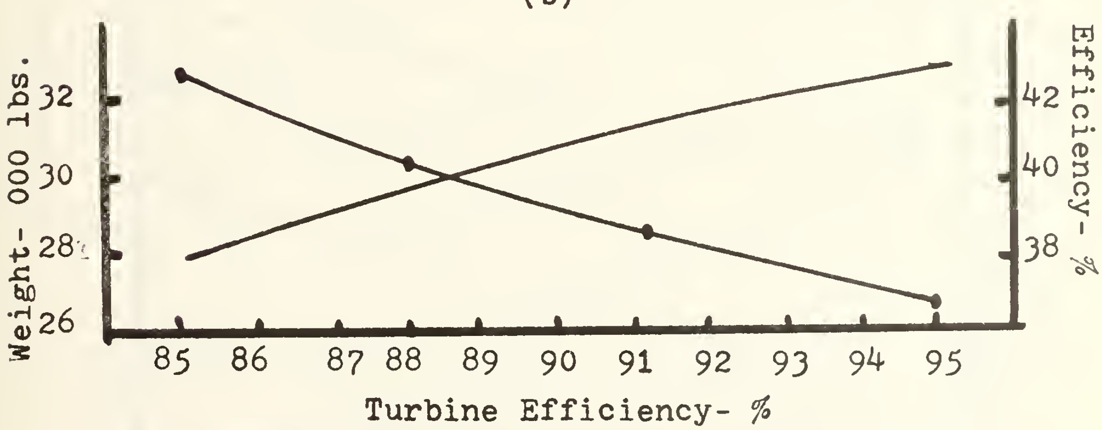
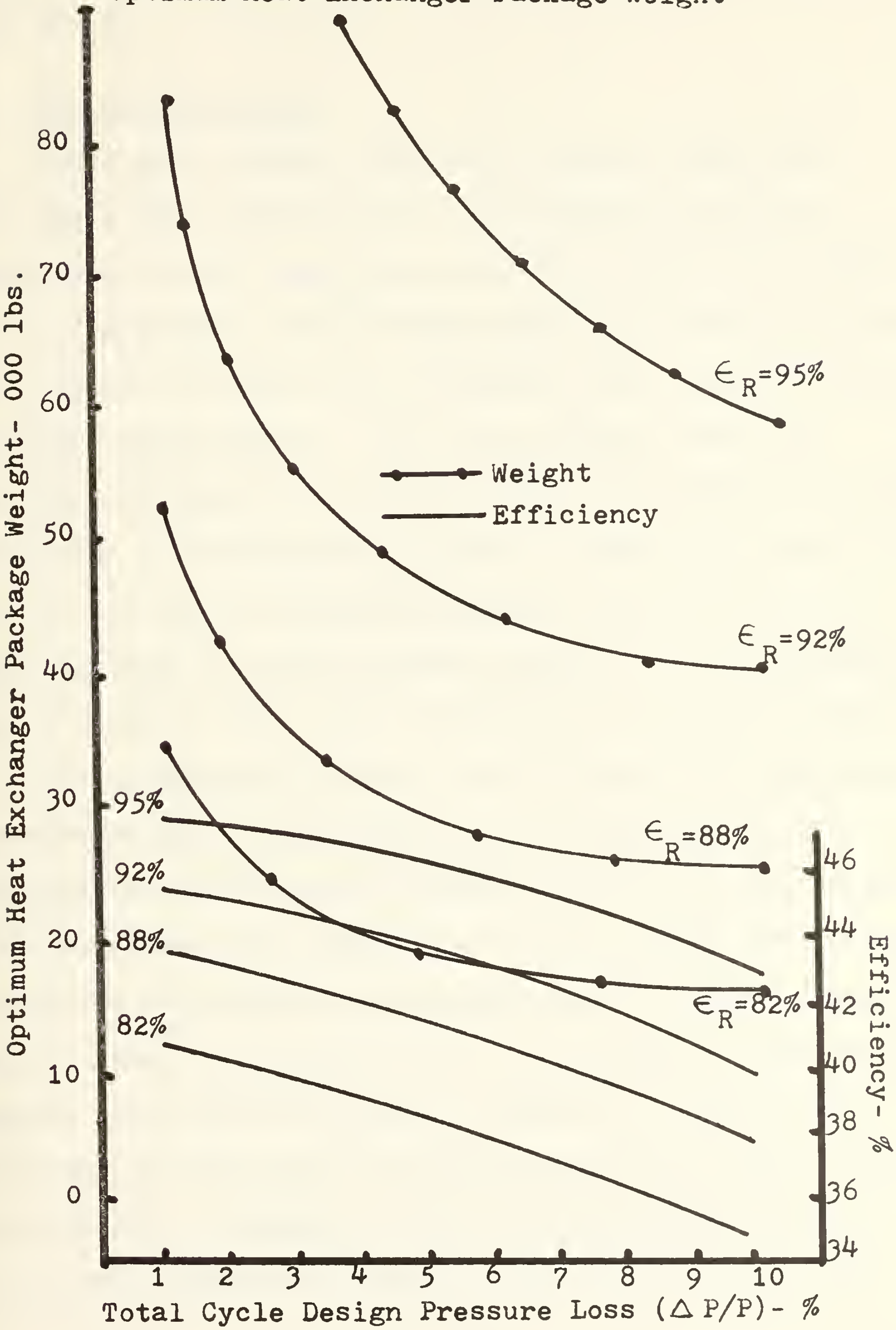


Figure 15

Influence of Total Cycle Design Pressure Loss on Optimum Heat Exchanger Package Weight



decreasing functions of ($\Delta P/P$). This trade-off will be determined by the mission endurance requirements for the ship design.

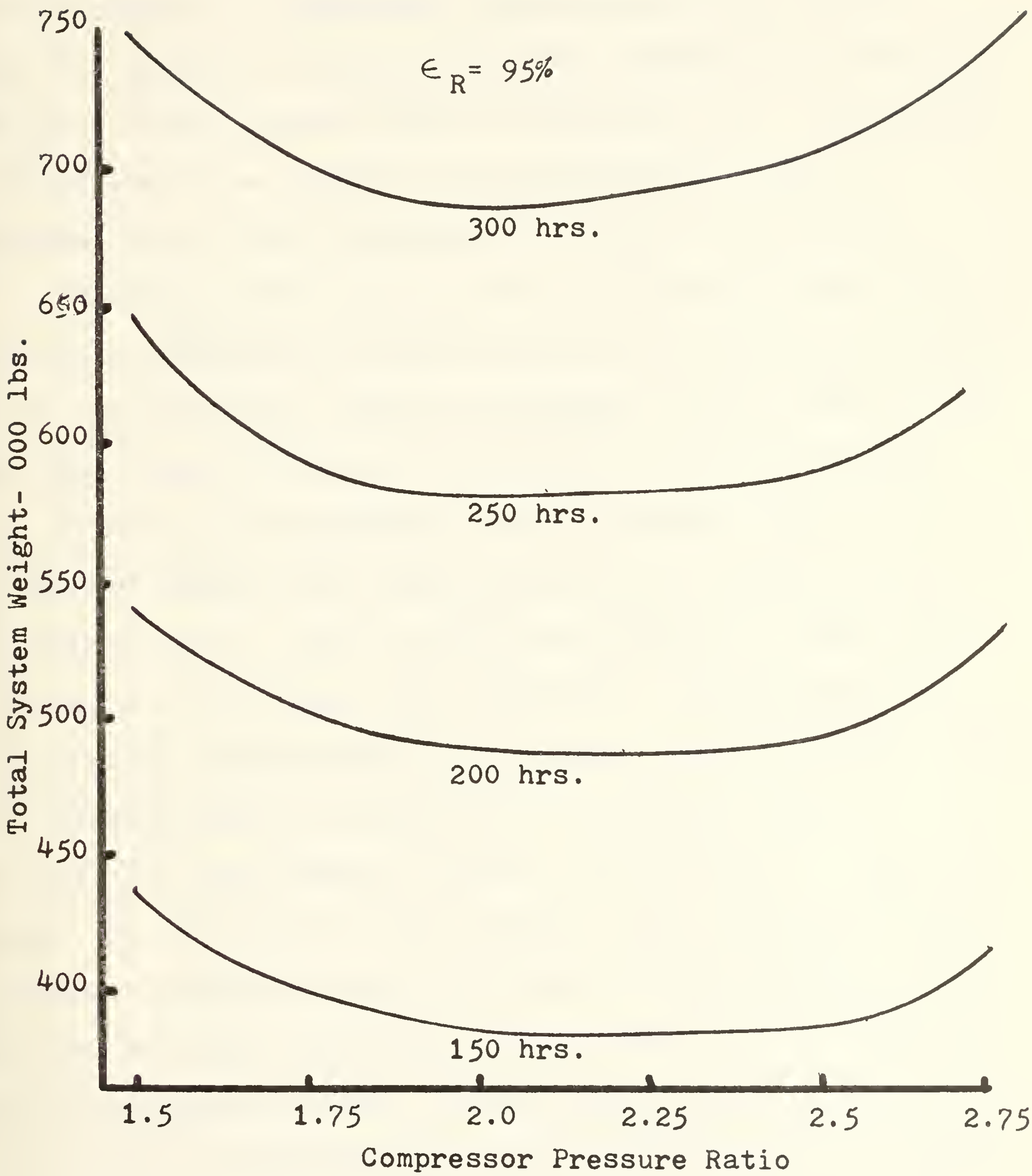
C. Design Trade-offs

Three major design trade-offs between weight and efficiency were identified in the parametric analysis. These were for the operating parameters compressor pressure ratio, regenerator effectiveness and cycle design pressure loss. Each of these can be examined based upon the mission endurance requirements. For this purpose a 6600 ton destroyer is chosen as baseline ship; its engineering plant consisting of two 40,000 HP helium, closed-cycle gas turbine units, each with an endurance power of 6000 HP per cycle. Based on this, the total system weight, ie. heat exchangers plus endurance fuel, can be determined at various endurances.

The trade-off in weight and efficiency for compressor pressure ratio is significant only at high levels of regenerator effectiveness because only at the high levels is there an appreciable difference between the pressure ratios at maximum efficiency and minimum weight. Even at the high levels, however, there are only modest weight variations between the optima, as shown in figure 16. Endurance durations of 150, 200, 250 and 300 hours are shown. The trend which is apparent is that minimum system weight occurs at a compressor pressure ratio which varies from

Figure 16

Weight-Efficiency Trade-Off for
Compressor Pressure Ratio



that for the minimum weight at low endurance hours, to that for maximum efficiency at high endurances. In other words, the fuel efficiency dominates the heat exchanger weight at high endurances. Therefore, choice of the "optimum" CPR at high levels of regenerator effectiveness will depend upon the endurance range requirement, whereas, that for the moderate effectiveness levels is generally not. At moderate effectiveness the designer could reasonably choose to operate at the CPR of maximum cycle thermal efficiency.

The same method can be used to analyze the trade-offs involving regenerator effectiveness and design cycle pressure loss. In figure 17, the heat exchanger weight (time 0) and the fuel weight increase with endurance hours is plotted for 80, 88 and 95% regenerator effectiveness. This shows a trade-off between the three effectiveness levels over the endurance hours. For endurance requirements of less than 100 hours at endurance speed (6000 HP), an 80% effective regenerator system would be preferred. For 100 to 210 hours, 88% effectiveness is favored and 95% effectiveness is favored for requirements greater than 210 hours. Figure 18 shows the variation of total system (fuel and optimum heat exchanger package) weight with cycle design pressure loss for 88% and 95% regenerator effectiveness at 150, 200, 250 and 300 endurance hours. Evident again is the preference for moderate effectiveness at 150 and 200 hours, but 95% at 300 hours endurance. Also shown is the optimum $\Delta P/P$ of

Figure 17

Weight-Efficiency Trade-Off for
Regenerator Effectiveness

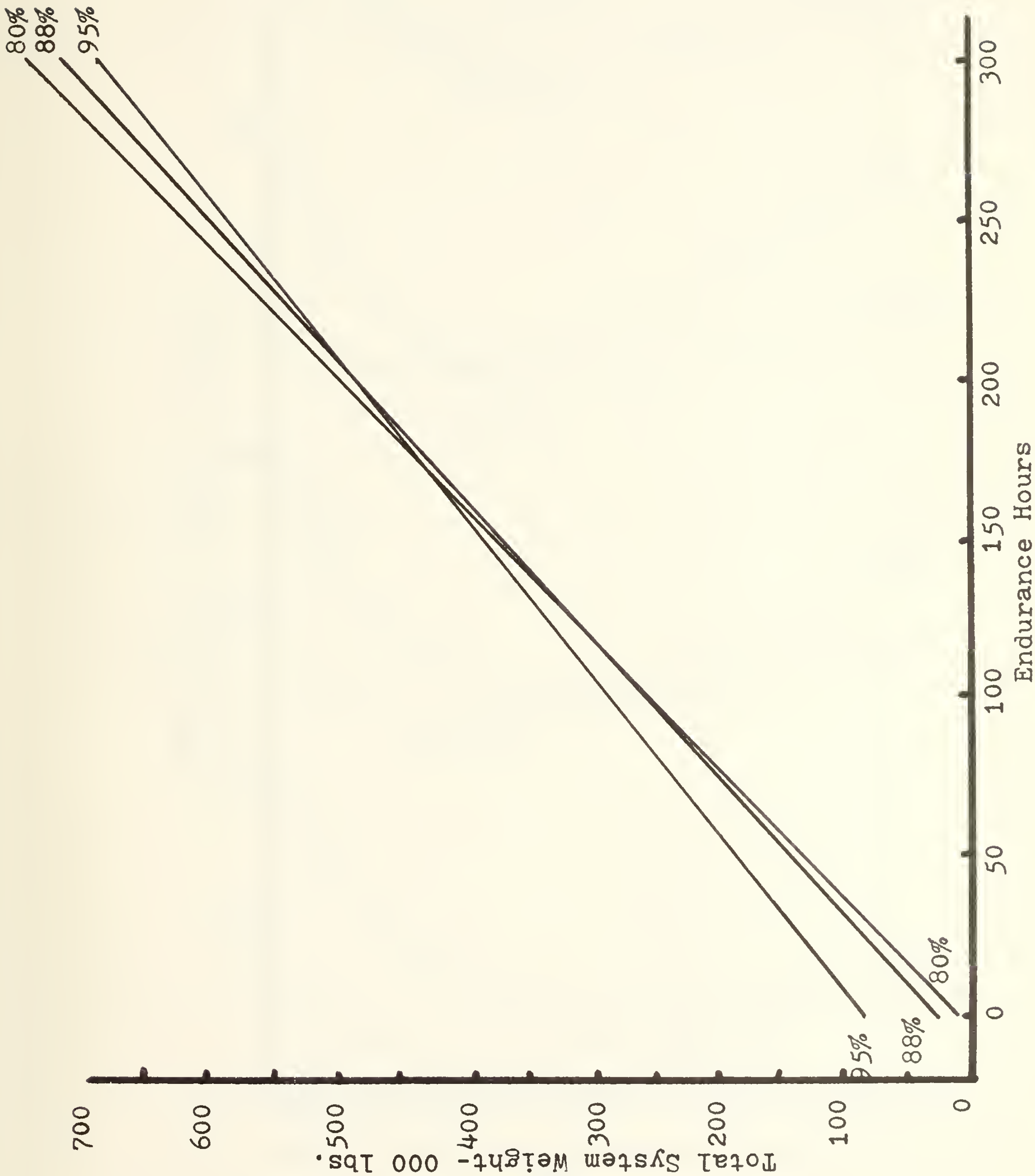
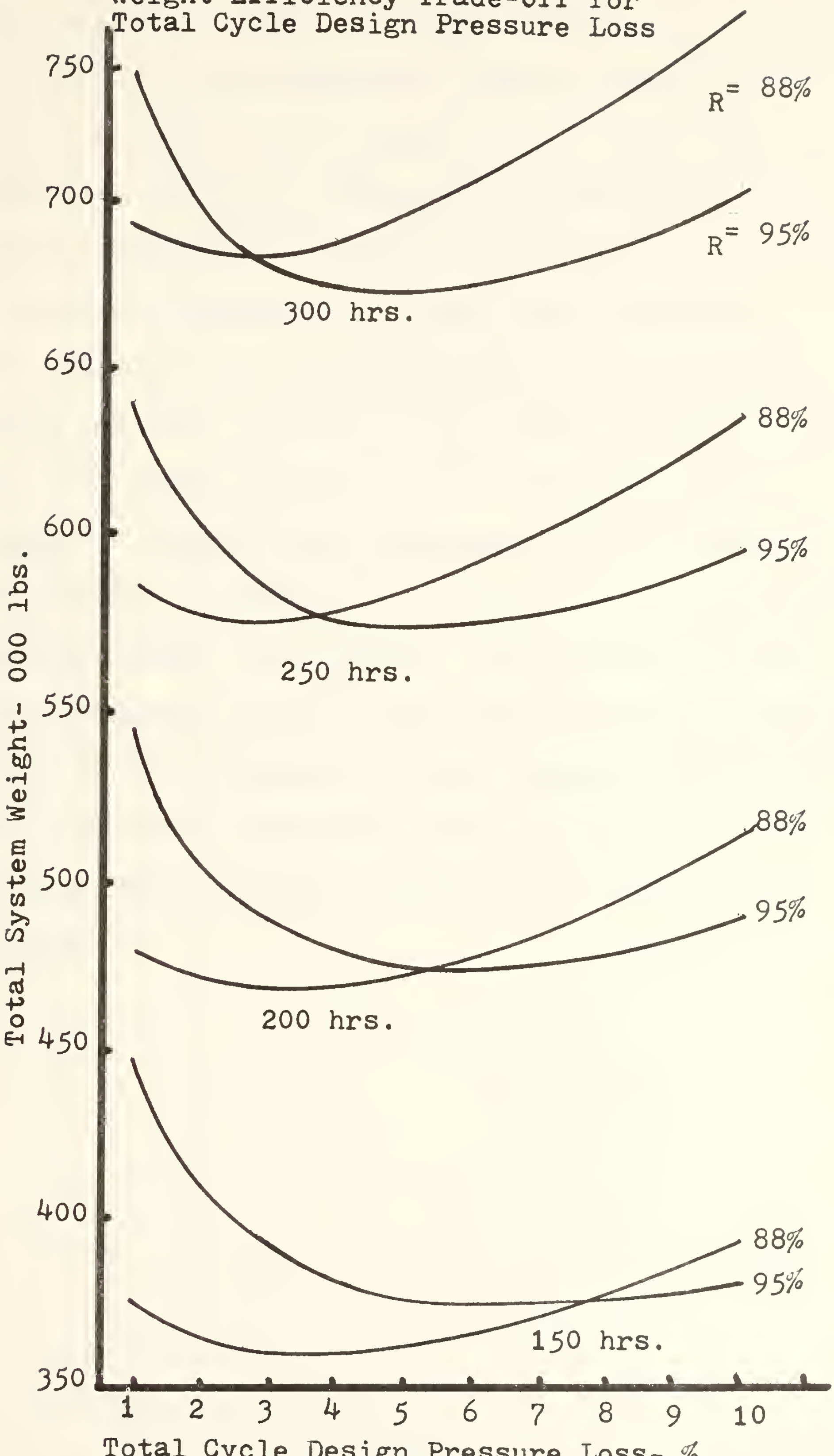


Figure 18

Weight-Efficiency Trade-Off for Total Cycle Design Pressure Loss



4% at 150 hours, 3% at 200 hours and 5% at 300 hours. At 250 hours endurance, there is an approximately equal system weight for 88% and 95% regenerator effectiveness at 2.5% and 5% cycle pressure loss, respectively. This would give indifference in choice of regenerator effectiveness based upon weight considerations alone. In practice, the designer is also bound by constraints on machinery arrangement. Since the regenerator is the dominating heat exchanger in both weight and size, it will be the prime concern of the designer. The design decision which may appear to be an indifference or even a slight advantage for 95% effectiveness based upon weight considerations alone, may in fact be determined by length constraints. For instance, at the apparent indifference point, the length of the 95% effective regenerator is approximately 35 feet compared to 8 feet for 88%. The designer's preference may well shift to 88% effectiveness because of the impact on the total ship system design.

CHAPTER V

SUMMARY AND CONCLUSIONS

The regenerative closed Brayton cycle was described and a detailed thermodynamic analysis presented. This served as a base upon which a means was developed for minimizing the system weight by minimizing the heat exchanger package weight for a given cycle. The weight minimization was based on an optimal allocation of the cycle design pressure loss to the heater, cooler and regenerator components. Single-pass, bare tube, shell-and-tube heat exchangers were assumed for the derivation. Using the optimum method developed, the impact of each of the heat exchanger sizing parameters and the major cycle operating parameters was examined and evaluated. From this analysis the following conclusions are made.

1. Heat exchanger weight is proportional to $\Delta P^{-0.41}$. For a given closed Brayton cycle, substantial weight savings can be achieved for the heat exchanger package by allocating the design pressure drop in an optimum manner. The optimum allocation scheme can be developed for the particular heat exchanger types and cycle parameters desired. The allocation scheme has been presented for shell-and-tube heat exchangers;

however, the basic procedure can be adapted to other heat exchanger types.

2. Heat exchanger sizing parameters are independent design variables specified by the designer. Prudent choice of these parameters, however, can have a marked effect upon the optimum system weight. The following conclusions are drawn.

a. The smallest practical tube diameter should be chosen consistent with basic manufactureability constraints, preservation of turbulent fluid flow and acceptable tube wall temperature. Generally the smallest tubes will be in the regenerator with larger tubes in the cooler and heater due to the turbulent flow and maximum wall temperature constraints, respectively.

b. The smallest practical tube wall thickness should be chosen consistent with manufactureability and strength requirements and ensuring acceptable tube wall maximum temperature, especially in the heater. Of the heat exchanger sizing parameters, the tube thickness parameter (D_0/D_1) has relatively the most significant impact on weights.

c. The smallest tube spacing should be used consistent with preservation of turbulent flow. Spacing is generally not a severe constraint in the cases of heaters and regenerators; however, the spacing ratio (s/D_0) is an important design tool in

designing saltwater coolers. In this case, larger s/D_0 may be used in order to increase saltwater flow velocity in the tubes to ensure turbulent flow and minimize marine growth while maintaining the optimum working fluid pressure drop in the shell and impacting weight only minimally.

d. To reduce weight, an equilateral triangular pitch tube arrangement should be used rather than a square pitch arrangement.

3. Basic cycle operating parameters also can have significant impact upon the optimum heat exchanger package weight. To some extent, the designer may have discretion in the level of some of these parameters, and prudent selection can result in weight savings. Based on examination of the major cycle operating parameters, the following conclusions are drawn.

a. For a cycle, there will be a compressor pressure ratio at which the cycle thermal efficiency will be maximum and the system weight will be at a minimum. Generally the compressor pressure ratio of maximum efficiency and minimum weight do not coincide. At moderate values of regenerator effectiveness (ϵ_R approximately 85%) both efficiency and weight reach broad optima; therefore, generally the maximum efficiency compressor pressure ratio can be selected. At high levels of regenerator effectiveness

(ϵ_R approximately 95%) there will be a design trade-off between high efficiency and low weight.

b. Cycle efficiency is not noticeably affected by the cycle pressure level. Weight, however, is a decreasing function of pressure level; therefore, the highest practical pressure level should be used.

c. Regenerator effectiveness has a great impact upon both weight and cycle efficiency. Increasing effectiveness results in a nearly linear increase in cycle efficiency, but it also gives an increasing weight and the rate of the weight increase accelerates greatly at high effectiveness levels. Therefore, a design trade-off will exist for the choice of regenerator effectiveness.

d. Increasing the turbine inlet temperature yields both improved efficiency and lower weight; therefore, the highest practical turbine inlet temperature will be determined by metallurgical considerations for the turbine materials. The primary impact of compressor inlet temperature lies in an increasing cycle efficiency for decreasing temperature. The weight effect is determined largely by the cooling water inlet and outlet temperatures. Generally, the lower the cooling water temperature and the lower the compressor inlet temperature, the better.

e. Increasing turbomachinery efficiencies yield

enhanced efficiency and weight; therefore, the highest machine thermal efficiencies should be used. The limit to the machine efficiency level will be determined by engineering considerations of complexity at high levels.

f. Increasing cycle design pressure loss results in decreases in both weight and efficiency. The weight effect is especially pronounced at high levels of regenerator effectiveness and low design pressure loss. A design trade-off will exist for selection of the design cycle pressure loss.

4. Assessment of design trade-offs for compressor pressure ratio, regenerator effectiveness and cycle design pressure loss can be made by examinimg the combined system weight effects of the heat exchangers and required fuel for the mission endurance of the ship design.

a. At high levels of regenerator effectiveness, the trade-off of compressor pressure ratio favors the minimum weight CPR at low endurance durations (150-200 hours at endurance horespower) and the maximum efficiency CPR for high endurances (250-300 hours at endurance horsepower).

b. A similar effect is found for regenerator effectiveness, where high effectiveness is favored at high endurance and low effectiveness at low endurance.

c. Optimum design pressure loss also depends upon

the regenerator effectiveness level. At high endurance requirements, the high regenerator effectiveness yields lower total system weight and visa versa at low endurance requirements.

d. Determination of regenerator effectiveness must not be based on weight considerations alone. The designer is also constrained by arrangement requirements, and, since the regenerator length and volume are highly dependent upon effectiveness, the regenerator size may override the choice of high effectiveness.

The conclusions from trade-off studies can be translated into a preference for low system weight for lower endurance requirements and high fuel economy for higher endurances. The weight of the fuel required for high endurance levels will be several times the weight of the system equipment.

5. The closed Braytom cycle affords the designer an opportunity to significantly reduce system weight through prudent selection of cycle operating parameters. The closed-circuit nature enables use of higher system pressure levels, yielding smaller components and higher efficiency. Also, inert gases can be used as working fluids permitting use of alloys with good high temperature properties but oxygen susceptibility, higher system temperatures and, consequently, higher efficiency and lower weight.

The ship system designer can thus "optimize" the closed Braytom cycle by first prudently selecting the cycle operating parameters in order to take advantage of the benefits afforded by the closed cycle. Use of high cycle pressures, high turbine inlet temperatures and high turbomachinery efficiencies, as well as "optimum" compressor pressure ratio, regenerator effectiveness and cycle design pressure loss as based upon the mission endurance requirements for the ship design, will yield enhanced performance and weight. Prudent selection of heat exchanger parameters, such as small diameter, thin-walled, closely-arranged tubes, will further these gains. Then, for the given cycle and heat exchanger parameters, the system weight can be further minimized through optimal allocation of the cycle design pressure loss by the method presentd.

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APPENDIX

A WORKED EXAMPLE

The following worked example is provided in order to illustrate the method for thermodynamic cycle calculations, heat exchanger sizing and optimum pressure drop allocation. Chosen for this purpose is a regenerative closed Brayton cycle with an air combustion loop and salt water cooler. Following are the cycle operating parameters (refer to figure 6).

Power Level	40,000 HP
Working Fluid	Helium
Compressor Inlet Temperature (T_1)	$540^{\circ}R$
Turbine Inlet Temperature (T_3)	$1960^{\circ}R$
Compressor Efficiency (η_c)	87%
Turbine Efficiency (η_t)	91%
Compressor Pressure Ratio (P_2/P_1)	2.3
Regenerator Effectiveness (ϵ_R)	88%
Combustion Loop Efficiency (η_b)	90%
Total Cycle Pressure Loss ($\Delta P/P$)	5%
Compressor Outlet Pressure (P_2)	400 psi.

<u>Cycle Calculations</u>:

Following the method of chapter II;

$$P_3/P_4 = (P_2/P_1)\left[1 - (\Delta P/P)_T\right] = 2.185$$

$$W_T = w\,\eta_t C_p T_3\left[1 - (P_4/P_3)^{\frac{\gamma-1}{\gamma}}\right] = W_E + W_c$$

For helium, $C_p = 1.241$ Btu/lb$_m$-^oR and $\quad = 1.66$.

$$W_t = 590.49w \text{ Btu/lb}_m$$

$$W_c = \frac{wC_pT_1}{\eta_c}\left[(P_2/P_1)^{\frac{\gamma-1}{\gamma}} - 1\right] = 301.87w \text{ Btu/lb}_m$$

$$W_T = 40{,}000 \text{ HP}$$

$$W_T = W_t - W_c = (590.49-301.87)w \text{ Btu/lb}_m$$

$$w = 98.0 \text{ lb}_m\text{/sec}$$

$$T_4 = T_3\left[1 - \eta_t(1 - (P_4/P_3)^{.397}\right] = 1484.2^oR$$

$$T_2 = T_1\left[1 + (1/\eta_c)\left[(P_2/P_1)^{.397} - 1\right]\right] = 783.2^oR$$

$$T_{2'} = T_2(1-\epsilon_R) + \epsilon_R T_4 = 1400.1^oR$$

$$T_{4'} \equiv T_4 - T_{2'} + T_2 = 867.3^oR$$

$$Q_H = wC_p(T_3 - T_{2'}) = 68{,}093.9 \text{ Btu/sec}$$

$$\eta_{total} = W_T/Q_H = 41.2\%$$

Heat Exchanger Calculations:

a. Regenerator:

The following assumptions are made for the regenerator calculations.

Tube inside diameter (D_1) .... 0.125 in.
Tube diameter ratio (D_r) 1.16
Tube spacing ratio (s/D_0) 1.35

Tube arrangement .... Triangular

Tube material .... Stainless Steel

High pressure helium in the tube

$$K_1 = \frac{0.174 \mu_1^{0.25} \left(\frac{4 w_1}{\pi} \right)^{1.75}}{\rho_1 g_0} = 11.71 \ \mathrm{lb_f\text{-}ft^{1.75}}$$

$$K_2 = \frac{0.174 \mu_2^{0.25} \left(\frac{4 \omega_2}{\pi} \right)}{\rho_2 g_0 D_r^{1.75} \gamma^3} = 22.94 \ \mathrm{lb_f\text{-}ft^{1.75}}$$

$$K_3 = \frac{0.022 k_1 P_{r_1}^{0.6}}{\mu_1^{0.8}} = 0.00378 \ \mathrm{Btu/{}^oR\text{-}lb_m^{0.8}\text{-}sec^{0.2}\text{-}ft^{0.2}}$$

$$K_4 = \frac{0.022 k_2 P_{r_2}^{0.6}}{\mu_2^{0.8}} = 0.00381 \ \mathrm{Btu/{}^oR\text{-}lb_m^{0.8}\text{-}sec^{0.2}\text{-}ft^{0.2}}$$

$$Q = 75{,}026.1 \ \mathrm{Btu/sec}$$

Because the tube and shell flow rates are equal in the regenerator, $T_{1m} = T_{h\text{-}out} - T_{c\text{-}in} = 84.1^oR$

$$K_5 = \left(\frac{\pi}{4 w_1} \right)^{0.8} \left(\frac{1}{K_3} \right) \left[1 + \left(\frac{K_3}{D_r K_4} \right) \left(\frac{w_1 D_r}{w_2} \right)^{0.8} \frac{q}{\gamma} \right]$$

$$q = 1.103 \ (s/D_0)^2 - 1 = 1.01$$

$$K_5 = 11.05 \ \mathrm{{}^oR\text{-}sec\text{-}ft^{0.2}/Btu}$$

$$n = \left(\frac{K_1 K_5 Q}{\pi \Delta T_{lm}} \right)^{0.513} \Delta P^{-0.513} D_1^{-2.026} = 178{,}148.4 \quad P^{-0.513} \ \mathrm{psi}^{0.513}$$

$$L = \left(\frac{K_5 Q}{\pi \Delta T_{lm}} \right)^{0.897} \left(\frac{\Delta P}{K_1} \right)^{0.103} D_1^{1.205} = 7.246 \quad P^{0.103} \ \mathrm{ft/psi}^{0.103}$$

$$K_6 = \frac{\pi \rho_t}{4} \left(D_r^2 - 1 \right) = 135.45 \ \mathrm{lb_m/ft^3}$$

$$K_7 = \left[\frac{\pi \rho_s P^* (1 + D_r q)}{2 \sigma_s} \right] = 40.86 \text{ lb}_m/\text{ft}^3$$

$$K_R = 1.1 (K_L + K_7) \left(\frac{K_5 Q}{\pi \Delta T_{em}} \right)^{1.41} K_1^{0.41} D_1^{1.13} K_P^{-0.41} = 27{,}040.0 \text{ lb}_m/\text{psi}^{.41}$$

$$K_p = 1.0$$

b. <u>Cooler:</u>

The following assumptions are made for the cooler calculations.

Tube inside diameter (D_1)	0.25 in.
Tube diameter ratio (D_r)	1.10
Tube spacing ratio (s/D_0)	1.75
Tube arrangement	Triangular
Tube material	90-10 Cu-Ni
Cooling water inlet temperature	520°R
Cooling water outlet temperature	580°R

Cooling water in the tubes

$$w_1 C_{p1} \Delta T_1 = w_2 C_{p2} \Delta T_2$$

$$w_2 = w_1 \left(\frac{C_{p1}}{C_{p2}}\right)\left(\frac{\Delta T_1}{\Delta T_2}\right) = 675.2 \text{ lb}_m/\text{sec}$$

$$K_1 = 1.74 \text{ lb}_f\text{-ft}^{1.75}$$

$$K_2 = 1.057 \text{ lb}_f\text{-ft}^{1.75}$$

$$K_3 = \frac{0.023 k_1 P_{r_1}^{0.4}}{\mu_1^{0.8}} = 0.002 \text{ Btu}/^\circ\text{R-lb}_m^{0.8}\text{-sec}^{0.2}\text{-ft}^{0.2}$$

$$K_4 = \frac{0.022 k_2 P_{r_2}^{0.6}}{\mu_2^{0.8}} = 0.00353 \text{ Btu}/^\circ\text{R-lb}_m^{0.8}\text{-sec}^{0.2}\text{-ft}^{0.2}$$

$$Q = 39{,}805.6 \text{ Btu/sec}$$

$$T_{lm} = 100.3^{o}R$$

$$K_5 = 16.17\ {}^{o}R\text{-sec-ft}^{0.2}/\text{Btu}$$

$$q = 2.38$$

$$n = 13{,}197.5\ \Delta P^{-0.513}\ \text{psi}^{0.513}$$

$$L = 13.83\ \Delta P^{0.103}\ \text{ft/psi}^{0.103}$$

$$K_6 = 92.03\ \text{lb}_m/\text{ft}^3 \qquad K_7 = 27.57\ \text{lb}_m/\text{ft}^3$$

$$K_C = 8462.2\ \text{lb}_m/\text{psi}^{0.41} \qquad K_p = 1.646$$

c. <u>Heater:</u>

The following assumptions are made for the heater calculations.

Tube inside diameter (D_1)	0.25 in.
Tube diameter ratio (D_r)	1.10
Tube spacing ratio (s/D_0)	1.35
Tube arrangement	Triangular
Tube material	High Temperature Alloy
Exhaust gas inlet temperature	$3317^{o}R$
Exhaust gas outlet temperature	$1800^{o}R$

Exhaust gas in the tubes

$$w_1 = w_2\left(\frac{C_{p1}}{C_{p2}}\right)\left(\frac{\Delta T_1}{\Delta T_2}\right) = 142.05\ \text{lb}_m/\text{sec}$$

$$K_1 = 50.12\ \text{lb}_f\text{-ft}^{1.75} \qquad K_2 = 15.92\ \text{lb}_f\text{-ft}^{1.75}$$

$K_3 = 0.000965\ \mathrm{Btu/^oR\text{-}lb_m^{0.8}\text{-}sec^{0.2}\text{-}ft^{0.2}}$

$K_4 = 0.004\ \mathrm{Btu/^oR\text{-}lb_m^{0.8}\text{-}sec^{0.2}\text{-}ft^{0.2}}$

$$Q = 68{,}093.9\ \mathrm{Btu/sec}$$

$$\Delta T_{lm} = 783.3^oR$$

$K_5 = 21.42\ \mathrm{^oR\text{-}sec\text{-}ft^{0.2}/Btu}$ $\qquad$ $q = 1.01$

$$n = 39{,}222.2\ \Delta P^{-0.513}\ \mathrm{psi}^{0.513}$$

$$L = 3.225\ \Delta P^{0.103}\ \mathrm{ft/psi}^{0.103}$$

$K_6 = 82.30\ \mathrm{lb_m/ft^3}$ $\qquad$ $K_7 = 39.72\ \mathrm{lb_m/ft^3}$

$K_H = 9587.2\ \mathrm{lb_m/psi^{0.41}}$ $\qquad$ $K_p = 3.15$

d. <u>Weight Optimization</u>:

$K_{8R} = K_2/K_1 = 1.959$ $\qquad$ $1 + P_H K_{8R}/P_L = 5.506$

$$\left[K_H/K_R(1+ P_H K_{8R}/P_L)\right]^{0.71} = 0.9527$$

$$\left[K_C/K_R(1+ P_H K_{8R}/P_L)(P_H/P_L)^{0.41}\right]^{0.71} = 1.8753$$

$$K_9 = 8.3340$$

$(\Delta P/P)^*_{R1} = 0.600\%$ $\qquad$ $(\Delta P/P)^*_H = 0.572\%$

$(\Delta P/P)^*_{R2} = 2.703\%$ $\qquad$ $(\Delta P/P)^*_C = 1.125\%$

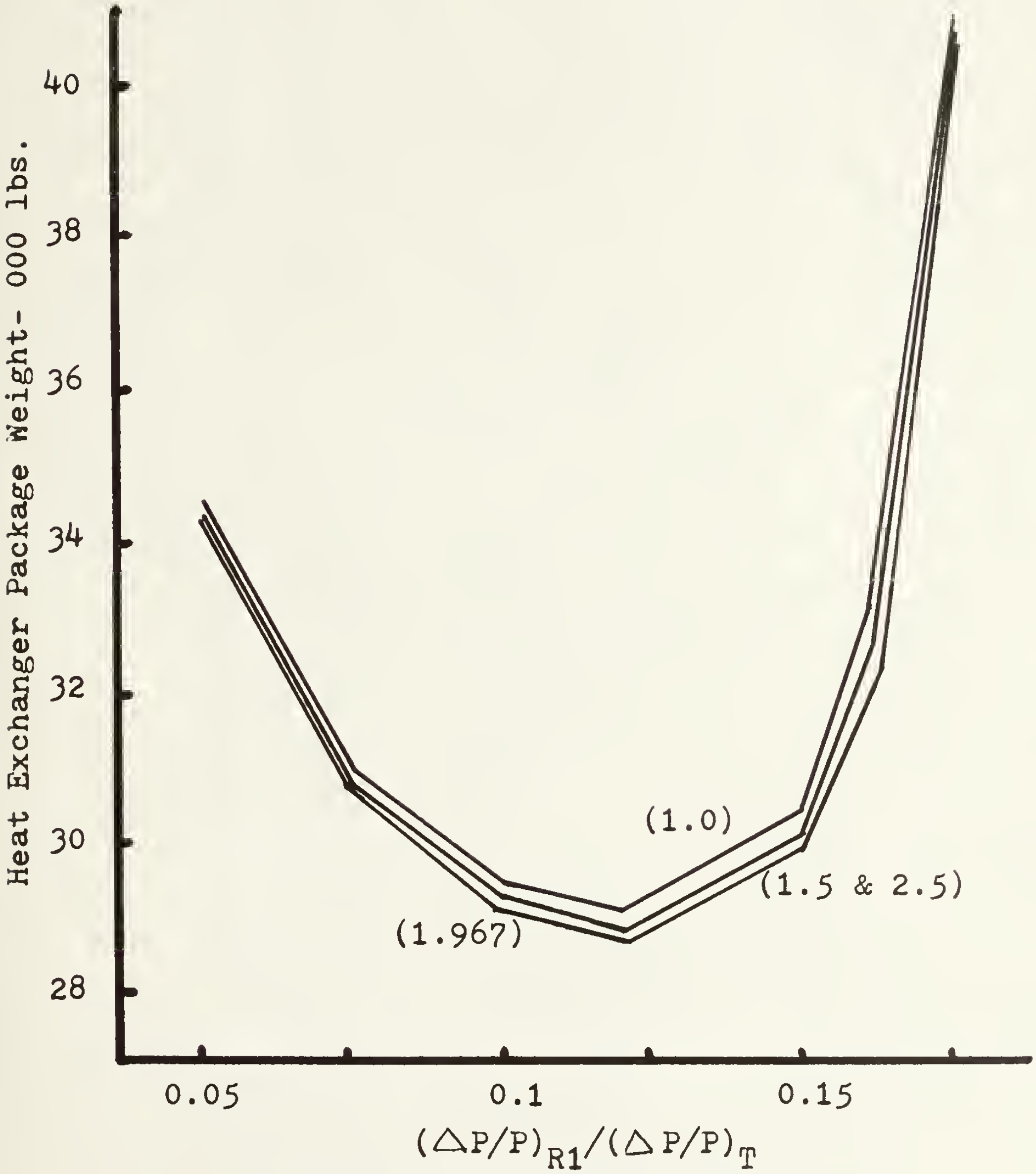
e. <u>Results:</u>

	ΔP	D_s-ft	L-ft	n	W- lbs.
Regenerator					
Tubes	2.4	5.18	7.93	113,693	18,885
Shell	4.7				
Cooler	2.0	3.83	14.82	9,354	3,267
Heater	2.3	4.85	3.51	25,653	6,427
Total	11.4 psi				28,579

The optimality of this solution is varified through figure 19 in which the heat exchanger package weight is plotted over a range of regenerator tube to total pressure drop, $(\Delta P/P)_{R1}/(\Delta P/P)_T$, for four compressor to heater pressure losses, $(\Delta P/P)_C/(\Delta P/P)_H$. The minimum weight point is the example solution.

Figure 19

Worked Example Optimality



( ) indicates value of $(\Delta P/P)_C/(\Delta P/P)_H$

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